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# Connection-based framework for assessing natural complexity in nonlinear adaptive systems

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## ABSTRACT

This study introduces a quantitative framework for assessing natural complexity in adaptive systems, based on connection measures weighted by sensitivity indices. The methodology integrates system modeling, sensitivity analysis, and complexity assessment, enabling continuous monitoring and decision support in dynamic environments. Natural complexity is defined as an optimal level at which the system behaves in accordance with its nature, sustaining coherence between structure and function. By employing sensitivity-weighted connections, the framework captures both internal organization and adaptive dynamics, overcoming limitations of traditional metrics such as Shannon entropy and fractal dimension, which often neglect interaction intensity and temporal variability. The framework is validated through two case studies: a computational model of an Intensive Care Unit and a real-world startup acceleration ecosystem. In the Intensive Care Unit, periods of overload were identified through peaks in complexity, associated with an increased number of highly sensitive parameter connections. In contrast, in the startup ecosystem, systemic idleness was reflected by lower complexity levels, driven by weakly influential interactions among actors. These findings highlight the responsiveness and interpretability of the proposed metric compared to conventional approaches, particularly in tracking adaptive states over time. This connection-based framework supports the management of adaptive information systems, offering a dynamic and scalable complexity assessment tool. Its applicability spans medical informatics, business management, and distributed systems optimization, providing real-time insights that improve resilience and efficiency. In addition, the approach aligns with industry 4.0 paradigms, facilitating preventive analyses and adaptive decision-making in advanced technological environments. By offering a unified methodology for complexity evaluation, this research advances understanding and control of complex adaptive systems.

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#### 1. Introduction

Science and engineering have a long history of efforts and methods focusing on how to study a system of interest by studying its parts. Many efforts focused on searching for the smallest structure that can characterize each system (and, ultimately, all systems) [1]. A different trend, starting around 1945, worked on the assumption that, in order to understand how a full system works, we need to know the laws and properties of its interactions [2,3]. Since then, research has evolved to try to understand how the combination of simple structures can result in complex compositions [4]. This shift in perspective led to the emergence of new research directions that aimed to understand how the combination of simple structures gives rise to complex systems.

Conceptually, complexity can be seen as the opposite of simplicity, where simplicity denotes clarity and predictability. In contrast, complexity often arises from incomplete knowledge about a system's governing principles or from the high-dimensional interactions within the system. In the context of systems, complexity could be due, in part, to ignorance of the principles or laws that rule each system. In other cases, even if the principles are known, the number of variables used to describe the system can be significant, to the point of generating emerging behaviors that are difficult to predict or control. Gell-Mann [5] claims that the apparent complexity is partially removed when we can find those laws.

When we analyze the concepts of simple/complex systems quantitatively, we can clearly see that simple systems normally consist of a single element, while complex systems include several components. These concepts can be applied to systems to demonstrate the statement that systems exhibit various types and levels of complexity [6], since they are made up of elements that act as a whole. According to Morin [7], complexity is the fabric of heterogeneous components that are inseparably associated. For this author, complex thinking uses the distinction/conjunction paradigm in knowledge organization, which conceives the unity and multiplicity of the real, escaping the abstract unity of holism and reductionism.

Complexity still needs to consider the balance between order and disorder [8], regularity and randomness [9]. Thus, an ideal complexity metric would intuitively need to treat completely random and ordered distributions as minimally complex, and intermediate configurations as highly complex [10]. These intermediate configurations are characteristic of complex systems, classified as physical and adaptive.

For Zadeh [11], all systems are adaptive, the real question is for what and to what extent. Thus, the main difference between the physical complex systems and the adaptive complex systems lies in the way in which the adaptation process influences such systems. While in physical systems there is a change in states, in adaptive systems the change occurs throughout a network of interactions. In general, the behavior of complex systems is characterized by self-organization in patterns, sensitivity to small variations in parameters, occurrence of rare events, and adaptive interaction, where agents learn and modify their strategies [4].

Sensitivity analysis attempts to understand how a system behaves due to variations in input parameters as follows: (i) how uncertainty in the input propagates through the system to contribute to the output variability and (ii) how the input and output parameters are correlated [12]. Sensitivity analysis methods allow us to measure how sensitive the system is to these variations and to associate internal aspects of the system with external factors. Both the internal organization and the external organization of each system contribute to the definition of complexity, as internal connections and their mechanisms reflect interactions with the environment [6].

The complexity of systems presents a challenge in various fields, including engineering, computer science, and business management. With the advancement of Information Systems (IS) and Information Technology (IT), quantifying complexity has become necessary to enhance performance and decision-making processes. Allaire et al. [13] propose an information-theoretic metric to measure the complexity of the system, while Holub [14] and Kaul et al. [15] emphasize structural approaches. The need for interdisciplinary strategies to manage complex systems is highlighted by Mora Tavarez et al. [16]. Xia & Lee [17] classify complexity into four dimensions, and Merali [18] suggests applying complexity science concepts to IS. Schütz et al. [19] employ entropy-based analysis to assess enterprise architecture, while Joseph & Marnewick [20] identify ten key elements influencing the complexity of the IS project.

Recent studies have advanced the development of quantitative metrics for measuring system complexity, with the aim of overcoming the limitations of traditional models. Various approaches propose complexity quantification based on connections, states, and behaviors [21–24], incorporating static and dynamic metrics that enable system comparison and the identification of critical elements [25]. These metrics have been applied to process models [26] and quality cost analysis [27]. Furthermore, Costa Junio [28] proposes a cost-based approach, integrating different complexity perspectives to improve decision-making processes.

The investigation of complexity in information systems encompasses various applications that influence both management and technological development. The complexity of IS development projects is analyzed through structural and dynamic dimensions [17], while complexity theory provides a conceptual framework to understand the emergent nature of systems [29]. Sensitivity analysis-based metrics are applied to medical management [30] and system modeling [22]. The complexity is also examined in the context of tourism and information technologies [31], as well as quantitative methods for manufacturing systems [32]. The concept of cross-complexity highlights the trade-offs between component simplicity and systemic complexity, providing the required information for complexity measurement and management [33].

Sensitivity analysis is necessary to investigate complexity, as some variables may emerge and have a significant impact on the system [23]. Therefore, when considering: (i) the increase in complexity of systems, (ii) the whole systems approach, (iii) complexity as a unifying variable, and (iv) the absence of a quantitative definition of practical and representative complexity, a gap becomes evident when synthesizing the descriptive and organizational characteristics of systems into a specific measure. In this context, we intend to investigate how the complexity of systems can be quantified, both in their individual components and as a whole.

**Table 1**Comparison of system complexity metrics.

Metric	Scope of measurement	Limitations	Specific contribution of $\psi(c, \gamma_c)$
Shannon entropy [40]	Measures uncertainty based on probability distributions	Does not account for system structure or dynamic interactions	Incorporates entropy with sensitivity-weighted connections
Fractal dimension [41]	Measures self-similarity and scaling in spatial or temporal patterns	Inapplicable to systems without geometric self-similarity	Focuses on sensitivity-weighted connections, independent of geometric patterns
Algorithmic complexity [42]	Estimates the minimum description length of a system	Computationally infeasible for large systems, ignores adaptability	Applies a tractable approach combining sensitivity analysis and connection structure
Degree centrality [43]	Assesses node importance based on the number of connections	Static view, does not reflect dynamic changes or parameter sensitivity	Dynamically weights connections according to sensitivity indices
Clustering coefficient [44]	Measures the tendency of nodes to form clusters within a network	Limited to structural properties, does not capture system behavior	Integrates both structure and behavior through sensitivity-weighted connections
Natural complexity $\psi(c, \gamma_c)$	Measures the complexity of adaptive systems through sensitivity-weighted connections	Dependent on sensitivity analysis and appropriate system modeling	Provides a reference level (natural complexity) for monitoring and system assessment

Despite the existing complexity metrics, a unified approach to defining an optimal reference level for system behavior is still lacking. Traditionally, system analysis has focused on process modeling and interactions, without a consolidated quantitative measure to express inherent system complexity. To address this gap, we introduce the concept of natural complexity  $\psi_n(c, \gamma_c)$ , defined as the optimal level of complexity at which a system operates according to its intrinsic rules and objectives. This framework facilitates the identification of key systemic patterns, providing a solid foundation for decision-making in dynamic environments.

This notion of an optimal operating condition also finds resonance in classical philosophical thought. According to Plato's thinking about occupations, all things are produced more plentifully and easily and of a better quality when one man does one thing that is natural to him [34]. Based on Plato's philosophical view, we define the natural complexity of systems as a specific level of complexity in which the system acts consistently with its nature. In other words, the system performs its functions properly and reaches its objectives. In a different perspective from Charbonneau [35], we apply the term natural complexity to all systems, natural and human-made. The concept provides a fair and reasonable reference for system analysis and monitoring: fair because the system performs as required according to established rules, and reasonable because it operates within its own limits, avoiding overload conditions.

Although the concept of natural complexity provides a functional and normative reference for system operation, existing complexity metrics, such as Shannon entropy, fractal dimension, and algorithmic complexity [36–38], they still present limitations in their ability to integrate structural and dynamic aspects of systems, particularly in adaptive contexts [22,39]. Table 1 summarizes the main characteristics and constraints of these metrics, highlighting the need for more comprehensive approaches. This comparative analysis provides the basis for introducing natural complexity, which integrates both dynamic and structural aspects of system behavior. The metric is derived from a connection-based representation of the system, in which each internal link is weighted according to its sensitivity index, quantifying the impact of input parameters on output behavior. This formulation enables the integration of connection density and temporal variability, offering a more comprehensive perspective compared to traditional metrics based solely on static structure or entropy.

In this work, we introduce a complexity metric based on weighted connections,  $\psi(c,\gamma_c)$ , utilizing sensitivity indices and entropy principles to quantify the complexity of the system. The metric uses sensitivity indices to weight internal connections within the system model, while entropy, calculated based on the second law of thermodynamics, defines the complexity measure. This metric enables estimation of the optimal complexity and serves as a reference to compare the behavior of the system under different conditions. The proposed approach is validated through two case studies: one related to the management of the intensive care unit (ICU) and another that involves the development of a startup within an acceleration ecosystem. This methodology has broad applicability in technological infrastructures, distributed systems, medical informatics, and business modeling, offering valuable insights for improving Information Systems.

Despite these contributions, a unified quantitative approach that integrates both structural and dynamic aspects of complexity, particularly in adaptive systems, remains underdeveloped. Existing metrics, such as entropy-based measures and topological descriptors, often provide only partial views of system behavior, overlooking the intensity and variability of interactions over time. To address this gap, the proposed connection-based framework models the system as a functional network in which each connection is weighted by the sensitivity of the outputs to the associated input parameters. This approach captures both the structural presence and the functional relevance of internal interactions, enabling the identification of adaptive states and structural bottlenecks. It is validated through two case studies, an ICU and a startup acceleration ecosystem, demonstrating its applicability in managing complex systems across domains.

The structure of this work is as: Section 2 presents the theoretical background, including systems, modeling, simulation, complex systems, and sensitivity analysis. Section 3 details the proposed methodology. Section 4 presents the results, which are discussed in Section 5. Finally, Section 6 summarizes the main findings and contributions.

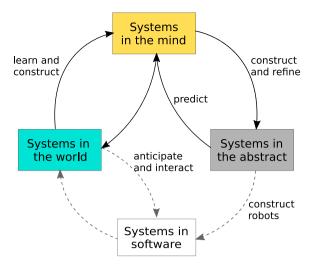


Fig. 1. Systems domains.

#### 2. Theoretical background

In this section, we introduce the fundamental concepts of systems and their models used in simulations. We begin by defining systems and models within the context of Systems Science, which explores features shared by various types of systems. We then focus on complex systems, examining their unique behaviors through sensitivity analyses to identify relationships and describe behavior patterns.

#### 2.1. Systems, models and simulations

A system is a set of organized interactive elements that relate to other entities and exist in a specific environment [2,6,22,45]. According to Maier [46], a system is a collection of components that produce a behavior or function that cannot be achieved by any individual component. Mobus and Kalton [6] classify systems into three domains: (i) systems in the world, (ii) systems in the mind, and (iii) systems in the abstract, referring to ontological, epistemological, and mathematical/symbolic aspects, respectively. The mathematical and symbolic representation of systems in reality occurs after mental abstraction. This concept is illustrated in Fig. 1, taken from Mobus and Kalton [6] and adapted from Calixto et al. [47].

The science of systems studies patterns and common behaviors in systems, regardless of the hierarchical level, and is classified as a formal science due to its mathematical foundation [48]. Its applications in physical, biological, cognitive, and other systems belong to phenomenological sciences [49] and relate to several fields of study, as illustrated in Fig. 2, taken from Calixto et al. [47]. While system science encompasses theory with mathematical definitions, system design encompasses practical methods for creating human systems, making it a normative science. System science proposes principles that govern all systems, including the concept of systemness introduced by Mobus and Kalton [6], which represents the recursive property of a system that includes smaller systems and is included in larger systems while maintaining its integrity.

The concept of totality in a system involves the internal cohesion of elements delimited by boundaries, which determine the flow of material, energy, and information with other systems [2,6,50]. Among the various types of systems, complex systems consist of interconnected entities whose behavior is governed by adaptable or non-adaptable rules [51]. These systems explore how internal relationships generate collective behaviors and how external relations are formed [52]. Complex systems can be categorized as physical or adaptive, each with distinct characteristics.

From simple operating rules, systems can exhibit complex collective behaviors, advanced information processing, and the ability to adapt through learning or evolution. These characteristics, as defined by Mitchell [53] and Holland [4], include self-organization into patterns, chaotic behavior, fat-tailed behavior, adaptive interaction, and emergent behavior. In complex systems, rare events occur more frequently than expected by the normal distribution. Adaptive interaction occurs when agents adjust their strategies based on experience. Examples include bird flocking patterns, fish schooling, the butterfly effect in chaotic behavior, and fat-tailed behavior in rare events. Adaptive interaction is also observed in mass extinction events and market crises. In all these cases, emergent behavior indicates that the system possesses properties that cannot be derivable from the simple sum of its parts, highlighting nonlinear interactions [4,22].

According to Baryam [54], complex collective behavior arises from specialized coordination of parts. These behaviors are categorized into individual behaviors as random, coherent and correlated, observable in physical, biological, and social systems, as illustrated in Fig. 3, adapted from Baryam [54]. When individual behaviors are random or coherent, collective behavior is simple. However, when behaviors are correlated, it becomes complex, combining regularity with randomness. Waldrop [55] argues that

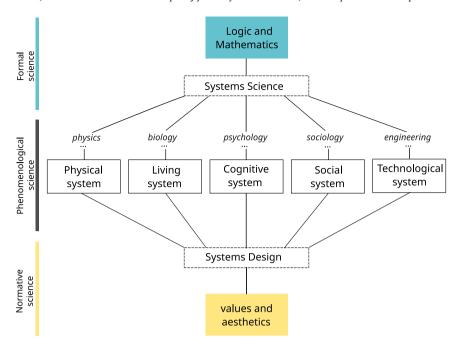


Fig. 2. Classification of sciences and systems.

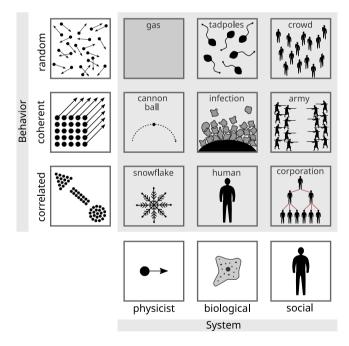


Fig. 3. Examples of individual system behaviors.

complex systems achieve a unique balance between order and chaos, incorporating features such as spontaneity, self-organization, adaptation, and activity. There are two main types of complex systems: physical systems, with fixed elements and laws where only the positions of the elements change over time, and adaptive systems, which are constantly changing. Physical systems exhibit self-similarity, scaling, network structure, and dynamics, observed in fractal curves that represent the continuous and regular repetition of geometric shapes [54].

Physical complex systems exhibit characteristics such as scaling, represented by power laws, regular or irregular network organization, and state-based dynamics [4]. Adaptive complex systems, on the other hand, consist of agents that learn and

adapt through interactions involving random variation and selection. The key difference between the two lies in how adaptation affects the system: in physical systems, state changes occur, whereas in adaptive systems, change permeates the entire network of interactions [4,5,11]. Holland [56] identifies specific features of adaptive complex systems, including parallelism, conditional action, modularity, and adaptation and evolution. These features can be computationally modeled, allowing the observation and manipulation of these systems using simulation software and high-performance hardware.

These unique features of adaptive systems, particularly the evolving structure of interactions and learning-based behavior, demand complexity metrics capable of reflecting both connection relevance and dynamism. This study addresses this need by proposing a metric that weights system connections based on sensitivity, allowing the representation of adaptation as a measurable variation in interaction patterns and system outcomes.

Models represent mental and abstract conceptions of systems. Creating models involves a process of simplification, separation, and identification, but is inherently uncertain, even when extensively tested [57]. The model cannot be validated as an exact representation of the system, since it always involves some form of abstraction from reality [58]. The same system can have different models depending on the level of abstraction and the selected components. Models can be associated with experiments and implemented as computer simulators. The accuracy of the simulator is assessed by applying tests that compare its behavior with the model's specification. Model validation occurs by comparing the simulator's results with those of the real system. The accuracy of both the simulator and the model can be evaluated with verification and validation techniques [59].

The modeling and simulation process, as described by Wainer [59], involves several steps: problem formulation, conceptual model design, observation and analysis of input and output data, and modeling itself. The problem formulation defines the system, identifies variables, performance metrics, and, if necessary, the objective function. Next, a conceptual model is created that describes the structure and behavior of the system. The data observation and analysis phase determines the sample size and the nature of the attributes before collecting the input and output data. These elements form the foundation of the modeling process.

Simulation involves implementing the model using a simulator and conducting experiments with techniques such as sensitivity analysis and optimization [59]. Analyzing the results provides insight into the behavior of the original system. Simulation models are classified into three categories: Monte Carlo simulation, continuous simulation, and discrete event simulation, each focusing on different types of variables over time [60]. Techniques for discrete or continuous modeling and simulation are categorized on the basis of the representation of model state and time variables. Time can be treated as continuous or discretized, and state variables can be represented as a continuous set or a finite set of values. Various techniques are used for each classification, including differential equations, bond graphs, difference equations, finite element method, discrete event system formalisms, and Petri nets, among others [59].

#### 2.2. System complexity

Throughout history, human organization has evolved from simple hierarchies with few members to complex network structures driven by the Neolithic, Industrial and Information revolutions. These transformations have directly influenced the control methods within groups, as illustrated in Fig. 4 (not to scale), adapted from Baryam [54]. In hierarchical control, the leader's behavior is replicated on a larger scale, whereas in distributed control, decisions are decentralized and made by interacting teams [54]. The rise of network organization in civilization is evident as the world becomes increasingly interconnected in economic, political, and social terms. According to Page [51], during periods of limited and distant interactions, the systems were more episodic, consisting of events or situations that occurred occasionally or sporadically, without a defined connection or pattern among them.

System complexity refers to the presence of multiple interconnected parts that follow rules and often adapt through learning, natural selection, or other methods [4,51,53]. Even simple rules can result in complex structures and behaviors. Holland [4] defines this characteristic as perpetual novelty, highlighting its prevalence in most complex systems, exemplified by DNA, which consists of combinations of only four nucleotides, although no two human beings are exactly alike. The complexity systems approach encompasses two main aspects: (i) the quality of what is considered complex and (ii) a scientific field with several areas of study [3,4,53,61,62]. Relevant characteristics of complexity include non-linearity, emergence, self-organization, diversity, interdependence, and evolution. The scientific field of complexity is marked by a series of influential studies [2,5,62–64].

The complexity of the system is manifested through prominent characteristics such as non-linearity, emergence, and self-organization. Nonlinearity means that linear relationships cannot be superimposed to describe the overall process [2]. Emergence occurs when complex behaviors arise from simple interaction rules [65], and as Holland [4] explains, it occurs when system elements combine to generate properties not found by merely summing the properties of individual elements. Self-organization is the ability of certain systems to establish their own structures and rules without external intervention [62].

Diversity and complexity are intrinsically related. Bak [62] defines complex systems based on their variability levels, observable on various scales. The author emphasizes that humans recognize each other due to their differences and notes that the brain is the most complex system because it creates representations of other complex systems. Interdependence refers to the mutual influence between parts and the whole. Removing a part of the system affects not only that part but also the entire system, demonstrating the strength of coupling among components [6,52]. Evolution is the ability of systems to follow developmental trajectories in space and time, applicable at all scales, including the subatomic level [6,45]. For complex organisms, evolution explains their formation through incremental changes determined by fitness properties [66].

To measure complexity, various metrics have been developed as tools for quantitative evaluation of the system, allowing comparisons between different configurations or different systems [45,53,67]. Lloyd [68] categorizes approximately 40 metrics, noting that the diversity of approaches represents variations on core themes, posing a challenge similar to describing electromagnetism before

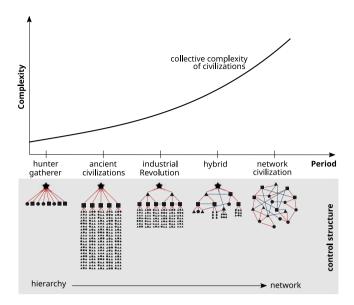


Fig. 4. Historical progression of human complexity and the corresponding control structures.

Maxwell's equations. Some metrics, such as Shannon entropy and algorithmic complexity, measure complexity based on the degree of randomness in the system. Therefore, the more random the organization, the higher the measured complexity [10,40,69]. However, true complexity lies in the balance between order and disorder, regularity, and randomness. Consequently, the ideal complexity metric treats completely random or completely ordered distributions as minimally complex, while intermediate configurations are considered highly complex [9,10,22].

## 2.3. Sensitivity analysis

Sensitivity analysis, according to Saltelli [70], investigates how specific inputs influence the output of a model, evaluating the relationship between uncertainties in the output and various sources of uncertainty in the inputs. The goal is to quantify the relative contribution of each input to the model output [71]. This analysis is often paired with an uncertainty assessment [72,73]. While uncertainty assessment assesses the degree of uncertainty of a specific conclusion, sensitivity analysis identifies the sources of that uncertainty [74,75]. In system research, it is required to simplify models, check the resilience of optimal solutions, and understand the relationships between input and output variables to manage different scenarios or circumstances. Sensitivity analysis is a critical tool in decision-making, communication, system understanding, and model development. Indicates how sensitive the system is to potential changes and errors in the inputs [76].

In sensitivity analysis, the approach for the input parameter search space can be local or global. Local analysis relies on point estimates of parameter values, while global analysis considers sensitivity based on the entire parameter distribution [77]. In the local approach, system outputs are evaluated by varying input parameters one at a time while keeping the others fixed at their central (nominal) values [22,78]. In global analysis, the input parameter space is explored within a finite (or feasible) region, and the outputs are obtained by averaging the variations of all input parameters. Fig. 5(a), adapted from Calixto et al. [47] e Paiva et al. [79], illustrates measurements related to one parameter at a time. In each cycle, one parameter is modified, while the others remain at their nominal or base values,  $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)$ . In this way, in the first cycle, only the value of parameter  $x_1$  is changed, in the second cycle, the value of  $x_2$  is modified, and so on until the nth cycle, in which only the value of  $x_n$  is varied.

The spider diagram is a commonly used visual method to analyze system sensitivity. This method shows the variation curves of the input parameter in relation to the output of each individual parameter [80], as illustrated in Fig. 5(b), adapted from Gomes [22]. Variations can be both negative and positive from the base case value of the parameter  $\alpha_j$ . When  $x_1 = \alpha_1, x_2 = \alpha_2, \dots, x_n = \alpha_n$ , the output of the system  $\beta$  is obtained, represented by  $y = f(\alpha_1, \alpha_2, \dots, \alpha_n)$ . Graphically, the output  $\beta$  is indicated by the central point where the sensitivity curves intersect. The base case  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$  typically represents the optimal solution obtained after an optimization process or the precise estimation of inputs by a system expert [76,81]. Due to operational or optimization considerations, parameters may have constraints regarding the size of the interval, presenting values close to the base case value [82]. The sensitivity analysis of the system parameters can be applied both to the model and to the simulation or to the real system [22].

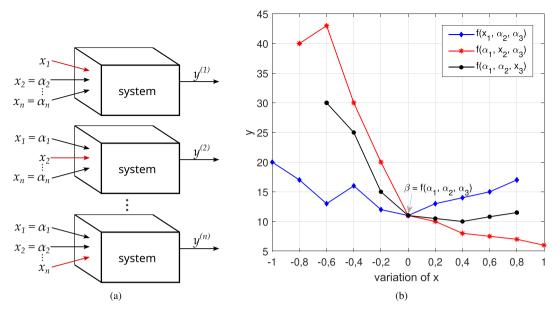


Fig. 5. Hypothetical example of: (a) measuring one parameter at a time for n input parameters and (b) a spider diagram with three parameters.

# 3. Methodology

This section addresses the proposed metric based on natural complexity for system analysis. The underlying premise is that each system has intrinsic complexity that ensures its effectiveness and efficiency. The text covers system analysis from the modeling phase to monitoring and subsequent decision making. We apply the proposed methodology considering the healthcare context and the analysis of a real SAE.

# 3.1. Systems analysis methodology based on natural complexity

Complexity metrics aim to quantify the complexity of a system based on a reference. Typically, we compare systems of the same type or different configurations of the same system. In the latter case, we can analyze these configurations in terms of their distance from a reference complexity value (or range) associated with ideal or fair performance conditions, which we define as the system's natural complexity. Although this study uses the metric  $\psi(c, \gamma_c)$  to assess such configurations, the concept of natural complexity is broader and not restricted to any specific formulation. The key requirement is that the metric used must adequately express performance variations under conditions such as overload or underuse. Fig. 6 illustrates a flow diagram for system analysis and monitoring based on natural complexity, using the complexity metric  $\psi(c, \gamma_c)$ .

The complexity metric  $\psi(c,\gamma_c)$  is obtained by simulations or experiments. Initially, we determine the value of natural complexity  $\psi_n(c,\gamma_c)$ , which corresponds to a system configuration established using optimization techniques (that is, an optimal or optimized solution) or by experts (best guess). Typically, in local sensitivity analysis, the base case  $\alpha=(\alpha_1, \alpha_2, \ldots, \alpha_n)$  refers to this configuration. Each parameter can then be modified from its base value up to  $\pm 100\%$  or within the limits of its range of viable values. In practical cases, defining the parameter variation interval means determining the system's operating ranges without compromising its physical limits.

Considering the system dynamics, Fig. 6 illustrates the flow for system analysis and monitoring, which comprises: (i) system modeling (Step 1 and Step 2), (ii) sensitivity analysis (Step 3 and Step 4), (iii) complexity analysis (from Step 5 to Step 8), and (iv) system monitoring and decision-making (after Step 8). The modeling involves model building (Step 1) and selecting input parameters and output variables (Step 2). Even in the case of experiments (in the real world), it is necessary to represent the system in terms of connections to apply the complexity metric. The model must contain internal connections related to the input parameters.

The next phase after modeling involves sensitivity analysis, which includes defining the base case  $\alpha$  (Step 3) and calculating the sensitivity indices  $S_{x_j}$  (Step 4) using either local or global analysis. For real system analysis, the local approach (around the base case  $\alpha$ ) is recommended because it requires fewer scenarios. When the model is computationally simulated, the global approach can be applied, eliminating the need to define the base case  $\alpha$  in Step 3.

Using the sensitivity indices  $S_{x_j}$ , we calculate the connection relevance values  $\gamma_c$  in Step 7. Prior to this, Step 5 involves identifying the connections c, and Step 6 defines the relationship between the parameters  $x_j$  and the connections c. The relevance of the connection  $\gamma_c$  is then calculated in Step 7 based on the sensitivity indices  $S_{x_j}$ . In Step 8, we calculate the complexity of the system  $\psi(c,\gamma_c)$ , concluding the complexity analysis phase. After calculating the system complexity, we verify whether the value obtained is equal to the natural complexity  $\psi_n(c,\gamma_c)$ . If it differs, this indicates a potential problem within the system. The decision-making

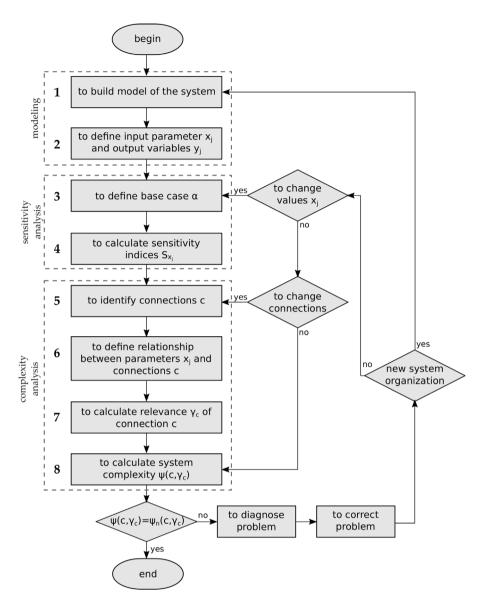


Fig. 6. Flow diagram for system analysis and monitoring based on natural complexity.

process to address the problem can involve changes to the organization, configuration, or set of possible connections to the system. Each type of change prompts a repetition of the process starting from: (i) Step 1 for new system organization, (ii) Step 3 for changing parameter values, and (iii) Step 5 for changing connections.

The changes implemented aim to restore the system to its natural complexity, which may correspond to a range of values associated with alternative configurations near the optimized solution or the base case defined by the expert. Although natural complexity  $\psi_n(c,\gamma_c)$  can be determined by optimization procedures or expert judgment, such approaches can vary considerably between domains and introduce subjective bias. To enhance generalizability and reduce dependence on subjective inputs, we propose combining heuristic knowledge with data-driven inference. For example, clustering techniques applied to simulation or historical data can reveal recurrent operational regimes characterized by high performance and structural stability.

In such cases,  $\psi_n(c,\gamma_c)$  can be defined not as a single fixed point but as a robust interval that reflects acceptable complexity levels. Furthermore, analyzing performance-to-complexity ratios  $(R_c)$  in the parameter space can help identify efficient operating plateaus. Sensitivity-based robustness assessments can further delineate a resilience region around the optimal configuration, reinforcing the interpretation of  $\psi_n$  as a fair and adaptable reference. Together, these strategies support the generalization of the concept of natural complexity in heterogeneous adaptive systems.

Within this context, local sensitivity analysis remains appropriate for verifying the resilience of the base case, as variations in input parameters may influence both system behavior and the relevance of internal connections  $\gamma_c$ . For this purpose, we adopt analytical and statistical methods to calculate sensitivity indices, which are then integrated into the system complexity metric  $\psi(c, \gamma_c)$ .

# 3.2. Sensitivity analysis metrics

Considering the local approach to the sensitivity analysis, we propose the area method and the local conditional variance method. These metrics are based on the spider diagram and the conditional variance, respectively. The area method uses the spider diagram to calculate the area between the curves and a line parallel to the abscissa axis, called the base axis. This line corresponds to a constant value on the ordinate axis equal to the output value of the base case scenario, when  $y = f(\alpha_1, \alpha_2, ..., \alpha_n)$ . The values of the input parameters  $\alpha_j$  are used as a reference for the local sensitivity analysis. While one parameter is modified, the others are kept fixed at their base value  $\alpha_j$ . Taking into account the known probabilistic distribution that governs each input parameter, random values of the parameters of interest can be generated. Thus, for the input parameters n and the scenarios m, which are the sets of input values, we have the matrix X, given by:

$$X = \begin{bmatrix} x_1^{(1)} & x_2^{(1)} & \cdots & x_n^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \cdots & x_n^{(2)} \\ x_1^{(3)} & x_2^{(3)} & \cdots & x_n^{(3)} \\ \vdots & \vdots & \ddots & \vdots \\ x_1^{(m-1)} & x_2^{(m-1)} & \cdots & x_n^{(m-1)} \\ x_1^{(m)} & x_2^{(m)} & \cdots & x_n^{(m)} \end{bmatrix}$$

$$(1)$$

In most cases, it is assumed that the input parameters are independent of each other, so the random values can be obtained separately from the distribution of each parameter. To obtain the output value corresponding to each set of input parameter values (rows of matrix X), the model simulation is executed m times, resulting in matrix Y given by (2). In this context, matrix X contains the values of the input parameters  $x_1$  to  $x_n$  for the scenarios m, and matrix Y contains the output values y corresponding to the sets of input parameters, called scenarios. The superscripted values indicate the scenario in question, as illustrated in Fig. 5(a). Once the input parameters are defined and the model or the system is at hand, the sensitivity analysis is carried out using methods such as the area method.

$$Y = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ y^{(3)} \\ \vdots \\ y^{(m-1)} \\ v^{(m)} \end{bmatrix}$$
 (2)

The application of the area method includes the following steps: (i) definition of the base case  $\alpha$ , (ii) simulation or experimentation using the one-factor-at-a-time method, (iii) construction of the spider diagram, (iv) definition of the analysis interval [a,b], (v) calculation of the area of polygons formed by the curve of each parameter and the base axis within the interval [a,b], and (vi) calculation of the sensitivity indices  $S_{x_j}^a$ , given by (3). In this equation,  $S_{x_j}^a$  represents the sensitivity index of the parameter  $x_j$  obtained using the area method,  $A_{x_j}$  is the area formed by the curve of the parameter  $x_j$  and the base axis, and n is the number of input parameters.

$$S_{x_j}^a = \frac{A_{x_j}}{\sum_{i=1}^n A_{x_j}}$$
 (3)

The sensitivity index  $S_{x_j}^a$  represents the contribution of the area of the parameter  $x_j$  relative to the total area of all parameters. Fig. 7 exemplifies the area delimitation in the spider diagram to calculate the indices  $S_{x_j}^a$ . The base axis corresponds to the dotted line. The areas  $A_{x_1}$  (in blue) and  $A_{x_2}$  (in gray) in Fig. 7 are established based on the interval of interest. By default, the input parameters range from -100% to +100% to analyze the sensitivity, Fig. 7(a). However, system operating restrictions can limit the extent of the parameter range, as observed in the parameter curve  $x_1$ . Since the indices  $S_{x_j}^a$  are relative values depending on the area, the interval [a,b] can be chosen considering the smallest range, as illustrated in Fig. 7(b).

Based on the conditional variance method [83], we propose a local sensitivity analysis in which the parameters of n-1 vary according to their probabilistic distributions and the parameter  $x_j$  remains constant at its base value. In this case, the sensitivity index results from the relationship between the output variance for fixed  $x_j$  and the unconditional variance when all parameters assume random values within their possible ranges. The term conditional refers to the condition of fixing one input parameter and observing the output variability, comparing it with the unconditional or total variance given by (4), obtained when all the input parameters n have varying values.

$$\sigma^{2}(Y) = \overline{\mu}_{\sigma^{2}}\left(Y|_{x_{\sim i}}\right) + \sigma_{\overline{\mu}}^{2}\left(Y|_{x_{\sim i}}\right) \tag{4}$$

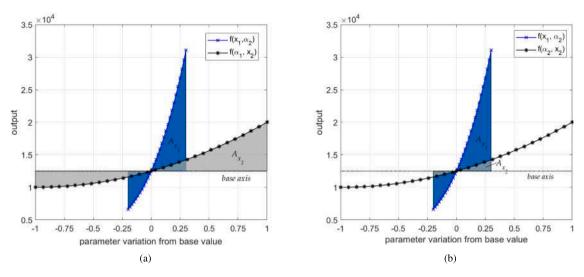


Fig. 7. Example of area delimited by the interval: (a) [-1,1] in the spider diagram and (b) [-0.2,0.3] in the spider diagram.

where  $x_{\sim j}$  corresponds to the variation of all input parameters except  $x_j$ ,  $\overline{\mu}\sigma^2$  is the mean of the variances,  $\sigma^2\overline{\mu}$  is the variance of the mean, Y is given by (2), and  $Y|x\sim j$  indicates the output vector when  $x_j$  is constant, which means  $x_j$  does not vary. The mean is used because the parameter under analysis assumes different values within its possible range; Fixing  $x_j$  at only one value would make the analysis dependent on that specific point. Thus, the sensitivity of the parameter analyzed is obtained by the ratio of the variance of the output means (for  $x_j$  fixed at different points) to the total variance, given by (5), where the  $\sigma^2$  superscript in  $S_{x_j}$  indicates the use of conditional variance to calculate the sensitivity index of the input parameters.

$$S_{x_j}^{\sigma^2*} = 1 - \frac{\sigma^2 \left( Y |_{x_{\sim j}} \right)}{\sigma^2(Y)} \tag{5}$$

where  $S_{x_j}^{\sigma^2*}$  is the sensitivity index of parameter  $x_j$  given by the local conditional variance method,  $\sigma^2$  is the variance, Y is given by (2), and  $Y|_{x_{-j}}$  indicates the output vector with  $x_j$  constant and the other parameters varying. Taking into account the sensitivity calculation in (5), the most sensitive parameter is the one that, when fixed, generates the largest average reduction in the output variance. The term average is used because the mean is calculated by fixing the parameter according to its distribution [83]. Since the analysis refers to an isolated parameter, it is called first-order analysis. If the input parameters are considered in pairs (or more), the analysis is second order (or higher orders), given by (6), where  $j, l \in \mathbb{N}^*$  with  $l \neq j \leq n$ . The term  $\sigma^2_{\delta_{x_j,x_l}}(Y)$  corresponds to the variance of Y due to the interaction between the input parameters  $x_j$  and  $x_l$ , resulting in part of the output y that cannot be obtained by the superposition of effects produced separately by  $x_j$  and  $x_l$ .

$$\sigma_{\overline{\mu}}^{2}\left(Y|_{x_{\sim j},x_{\sim l}}\right) = \sigma_{\overline{\mu}}^{2}\left(Y|_{x_{\sim j}}\right) + \sigma_{\overline{\mu}}^{2}\left(Y|_{x_{\sim l}}\right) + \sigma_{\delta_{x_{i},x_{l}}}^{2}(Y) \tag{6}$$

Although local sensitivity metrics are appropriate for monitoring variations around a known base configuration, their applicability becomes limited in systems that exhibit strong nonlinearities, discontinuities, or multiple equilibria. In such contexts, the response surface may contain bifurcation points or instability regions that are not detectable through local perturbations. Representative examples include climate models with tipping points, financial systems undergoing abrupt phase transitions, and biological regulatory networks characterized by multi-stability. In these scenarios, local analysis can provide misleading information by underestimating the influence of parameters whose effects emerge only in broader regions of the parameter space.

Accordingly, the local sensitivity analysis methods adopted in this study should be interpreted as tools for evaluating resilience and responsiveness near the operational point  $\alpha$ . They are not designed for comprehensive exploration of global system dynamics, but rather for incremental diagnostics under relatively stable conditions. The complexity metric  $\psi(c,\gamma_c)$  inherits this contextual constraint: when constructed from local sensitivity indices, it is well suited for real-time monitoring and operational validation, but it may require supplementation with global sensitivity techniques when investigating large-scale behavioral regimes or conducting robustness assessments.

# 3.3. Systems complexity metric

The proposed system complexity metric is expressed as a function of the connections c and their respective relevance  $\gamma_c$ , as defined in Eq. (7). The formulation of  $\psi(c,\gamma_c)$  integrates a relevance component  $\gamma_c$  with an entropy-like term,  $-P(c)\log_2 P(c)$ , inspired by information theory to represent both uncertainty and distributional diversity across the system.

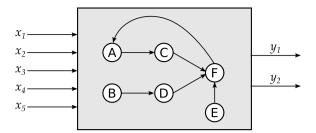


Fig. 8. Example of a model with five input parameters and two output variables.

This entropy-based component is not merely intended to quantify randomness or heterogeneity, but to capture the distributional balance of functional contributions throughout the system. In adaptive systems, a certain level of variability in the activation of internal connections is essential for resilience and responsiveness to environmental changes. If the system becomes overly regular, dominated by a small set of highly relevant connections, it may lack adaptability. Conversely, if all connections are equally and randomly active, the system tends to lose coherence. The entropy term models this trade-off, assigning lower complexity to configurations that are excessively uniform or excessively chaotic.

Accordingly, the natural complexity  $\psi_n(c,\gamma_c)$  corresponds to a balanced operational state in which the system maintains a structured yet diverse set of relevant connections. This configuration ensures both functionality and adaptive capacity. This interpretation aligns with theoretical perspectives suggesting that functional stability in complex adaptive systems depends on a controlled degree of unpredictability, a condition quantitatively expressed by the entropy term embedded in  $\psi(c,\gamma_c)$ .

In this context,  $\rho$  denotes the number of active connections at a given time,  $\gamma_{c_r}$  represents the relevance of the  $\kappa$ th connection, and  $P(c_k)$  is the probability of its occurrence. The relevance  $\gamma_c$  reflects the degree to which environmental variations influence the internal organization of the system. Calculated by identifying the input parameters associated with each connection and adding their respective sensitivity indices.

$$\psi(c,\gamma_c) = \sum_{\kappa=1}^{\rho} \left[ \gamma_{c_{\kappa}} - P(c_{\kappa}) \cdot \log_2 P(c_{\kappa}) \right]$$
 (7)

Fig. 8 illustrates an example of a model represented by a graph (internal organization), with input parameters  $x_1 \cdots x_5$  and outputs  $y_1$  and  $y_2$  (external organization). In this case, if the connection between the components A and C is directly related to the parameters  $x_1$ ,  $x_4$ , and  $x_5$ , then  $\gamma_{c_{AC}}$  equals the sum of the sensitivity indices  $S_{x_1}$ ,  $S_{x_4}$ , and  $S_{x_5}$ . Complexity  $\psi(c,\gamma_c)$  synthesizes the organization and behavior of the system into a unified measure through the weighted connections between its components in various configurations during operation. These connections are weighted by the sensitivity of the parameters, mapping the system behavior given the input-output relationship. Thus, complexity is the totality of the interacting system, representing its ability to function as a whole with its own internal dynamics and environmental interactions. Just as organization depends on structure, complexity depends on totality, adhering to the principle of systemness.

In the study of system complexity, the calculation  $\psi(c,\gamma_c)$  requires the values of the number of active connections  $\rho$  at each instant and the probability of connection P(c), defined according to the modeling performed. In the simulation of discrete events,  $\rho$ can be determined by (8) and P(c) by (9), where  $n_e$  is the number of entities,  $n_r$  is the number of resources, and  $n_a$  is the number of queues. In (9),  $n_e$  can be generalized as a function e(v), where constraints v can prevent connections between entities and resources or queues, reducing the possibilities of relationships. The function e(v) can take values in the range  $1 \le e(v) \le n_e$ .

$$\rho = \sum_{\kappa=1}^{\lambda} n_{c_{\kappa}} \cdot n_{\varepsilon_{\kappa}}$$

$$P(c) = \frac{1}{n_{e} \cdot (n_{r} + n_{q})}$$
(8)

$$P(c) = \frac{1}{n_e \cdot (n_r + n_q)} \tag{9}$$

We provide a hypothetical example to explain how to calculate the proposed complexity quantifier. First, it is necessary to recap the steps of the flow diagram in Fig. 6, from Step 1 to Step 8. Then, we explain how to carry out each step. We start by building the system model, which can be a computational model or just a connection model applied to a real system. The model can be represented by a graph, as illustrated in Fig. 8. Next, we define the input parameters  $x_1$ ,  $x_2$ ,  $x_3$ ,  $x_4$ ,  $x_5$  and the output variables  $y_1$ and  $y_2$ . When applying the local sensitivity analysis, we must define the base-case scenario  $\alpha$ . In this example, the base case refers to the configuration where  $x_1 = \alpha_1, x_2 = \alpha_2, ..., x_5 = \alpha_5$ . The base case  $\alpha$  usually refers to an optimal or optimized solution or the best guess provided by the specialists.

We then move to Step 4, where we calculate the sensitivity indices  $S_{x_i}$ . We choose a sensitivity analysis method, such as the area method, to calculate these indices. To apply the area method, we verify the impact on the output variable  $y_1$  when the parameter  $x_1$  is modified, while other parameters are fixed at their base values. This process is repeated for  $x_2$ ,  $x_3$ , etc., while other parameters are kept fixed at their base values. In Step 5, we identify the connections c between the elements of the system. Based on the model in Fig. 8, we identify six connections:  $c_{AC}$ ,  $c_{CF}$ ,  $c_{BD}$ ,  $c_{DF}$ ,  $c_{FA}$ , and  $c_{EF}$ . Step 6 involves defining the relationship between the parameters  $x_j$  and the connections c. The connections  $c_{AC}$  and  $c_{CF}$  are directly related to the parameters  $x_1$ ,  $x_4$ , and  $x_5$ . The connections  $c_{BD}$  and  $c_{DF}$  are directly related to the parameters  $x_2$  and  $x_3$ . The connection  $c_{FA}$  is directly related to parameter  $x_5$ , and  $c_{EF}$  is directly related to all parameters.

Step 7 To calculate the relevance  $\gamma_c$  of each connection c: The relevance values  $\gamma_{c_{AC}}$  and  $\gamma_{c_{CF}}$  are equal to the sum of the sensitivity indices  $S_{x_1}$ ,  $S_{x_4}$ , and  $S_{x_5}$ . The relevance values  $\gamma_{c_{BD}}$  and  $\gamma_{c_{DF}}$  are equal to the sensitivity indices  $S_{x_2}$  and  $S_{x_3}$ . The relevance  $\gamma_{c_{FA}}$  is equal to the sensitivity index  $S_{x_5}$ , and the relevance  $\gamma_{c_{EF}}$  is equal to the sum of the sensitivity indices  $S_{x_1}$ ,  $S_{x_2}$ ,  $S_{x_3}$ ,  $S_{x_4}$ , and  $S_{x_5}$ . Step 8 To calculate the system complexity  $\psi(c,\gamma_c)$ : We determine the number of active connections  $\rho$  at each instant to calculate the complexity of the system  $\psi(c,\gamma_c)$  using (7). For example,  $\rho$  is equal to 3 when the connections  $c_{AC}$ ,  $c_{CF}$ , and  $c_{FA}$  are active. The probability  $P(c_K)$  can be an experimental or theoretical value, defined based on the analysis of the system. Taking into account the dynamics of the system, the overall complexity  $\psi(c,\gamma_c)$  is computed as the average of the instantaneous complexity values in all time steps.

Since this is a methodological section, no numerical values are presented for the sensitivity indices or the complexity metric. The goal is to hypothetically illustrate the computational steps required to derive the proposed metric  $\psi(c,\gamma_c)$  while preserving the generality of the approach. Concrete applications, including real data and quantitative results, are detailed in the Results section for both the simulated ICU system and the real-world startup acceleration ecosystem (SAE), enabling the reader to trace the full procedure with empirical grounding. The steps involved in computing  $\psi(c,\gamma_c)$  are summarized in the flow diagram shown in Fig. 6, which guides the monitoring and analysis of adaptive systems. These steps are formalized through a pseudocode representation, Algorithm 1, which emphasizes the iterative and modular structure of the proposed method for computing system complexity. Unlike purely topological metrics, which only account for the existence of connections,  $\psi(c,\gamma_c)$  integrates behavioral sensitivity, making it suitable for adaptive systems in which the importance of each link varies over time. This dual perspective allows the metric to reflect not only the organization of the system but also the influence of parameter dynamics on its functional performance.

# **Algorithm 1** Calculation of system complexity $\psi(c, \gamma_c)$

```
Require: Input parameters \{x_1, x_2, \dots, x_n\}, output variables \{y_1, y_2, \dots\}, system graph structure
 1: Define base-case scenario \alpha = \{\alpha_1, \alpha_2, \dots, \alpha_n\}
 2: Select a sensitivity analysis method (e.g., area method)
 3: for each input parameter x_i do
 4:
        Compute sensitivity index S_x based on variations from \alpha_i
 5: end for
 6: Identify connections c_k in the system
 7: for each connection c_k do
        Identify input parameters \{x_{j_1}, x_{j_2}, \dots\} influencing c_k
 8:
        Compute relevance \gamma_{c_k} = \sum \hat{S}_{x_k}
 9:
        Estimate probability \hat{P}(c_k) (empirical or theoretical)
10.
11: end for
12: Determine number of active connections \rho at each time step
    for each time step t do
        Compute instantaneous complexity \psi_t = \sum_{k=1}^{\rho_t} \left[ \gamma_{c_k} - P(c_k) \cdot \log_2 P(c_k) \right]
16: return Average system complexity \psi(c, \gamma_c) = \frac{1}{T} \sum_{t=1}^{T} \psi_t
```

Traditional complexity measures, such as Shannon entropy or structural connectivity indices, provide partial views of system behavior. The proposed metric  $\psi(c, \gamma_c)$  advances these approaches by combining topological configuration with sensitivity-weighted relevance, thus capturing adaptive responses under varying operational conditions. This hybrid nature distinguishes it from static or only structural metrics.

This measure integrates both the organizational structure and the behavior of the system, since the connections are weighted by the sensitivity of their corresponding parameters. Such sensitivity-weighted connections capture not only the structural presence of links but also their functional significance under varying operational conditions. Developed from sensitivity indices, these weights enable the model to represent how variations in input parameters propagate through the system, supporting a dynamic, function-oriented complexity assessment.

In this context, the concept of natural complexity refers to the reference level of  $\psi(c,\gamma_c)$  expected under ideal or fair operating conditions, when performance is not limited by resource overload or underuse. Although the present study employs  $\psi(c,\gamma_c)$  as a quantifier, the notion of natural complexity is broader and can be captured by any metric capable of expressing coherent transitions between different operational regimes. In applied settings such as ICUs, the natural complexity can correspond to a configuration where patient flow, staffing, and equipment usage are balanced. Deviations from this state, such as excessive idle capacity or overcrowding, would manifest themselves as changes in complexity, allowing the early detection of inefficiencies.

Ultimately, system complexity is understood here as the emergent expression of a system that functions in interaction, as a whole. Although system organization depends on structure, complexity results from the integration of structure and behavior, aligning with

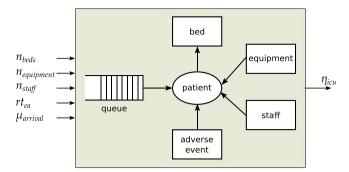


Fig. 9. Model of the ICU system comprising connections, input parameters, and output variable.

the principle of systemness. In this sense, natural complexity refers to the system's ability to maintain this integration under fair and efficient conditions. A suitable metric should reflect the transitions between underuse, optimal function, and overload, providing an interpretable measure of whether the system is operating near its natural complexity.

Building on this perspective, the complexity metric  $\psi(c,\gamma_c)$  is also applicable to real-world systems, provided that sufficient empirical records of input parameters and corresponding output values are available. In such contexts, sensitivity indices can be derived from observational data, enabling the construction of relevance-weighted connections without relying solely on simulations. This capability allows for the implementation of automated complexity tracking in operational environments. With continuous monitoring and real-time computation of  $\psi(c,\gamma_c)$ , alerts could be issued when the complexity of the system falls outside its expected natural range, supporting early identification of inefficiencies or abnormal conditions.

#### 4. Results

This section presents the results of applying the proposed methodology to two adaptive systems: a computational model of an Intensive Care Unit (ICU) and a startup acceleration ecosystem (SAE). For each case, we evaluate natural complexity using the connection-based metric  $\psi(c, \gamma_c)$  and compare its performance with conventional complexity metrics, including Shannon entropy, fractal dimension, algorithmic complexity, degree centrality, and clustering coefficient. The results highlight the ability of the metric to capture critical adaptive states, such as overload and idleness, and provide insight into system behavior under varying operational conditions.

To support the calculation of  $\psi(c,\gamma_c)$ , which requires sensitivity indices as inputs, different sensitivity analysis methods were used according to the specificities of each case study. In the ICU case, the area method was adopted because of its ability to graphically represent the local effects of parametric variations on system performance. In the SAE case, two complementary methods were applied: the area method, which facilitates interpretive visualization of individual influences, and the local conditional variance method, which quantifies the relative contribution of each parameter based on the reduction in output variance. The methodological choices were aligned with the nature of the available data and the analytical goals of each scenario.

# 4.1. Case study: complexity of an intensive care unit

In the healthcare sector, critical or potentially serious patients are commonly treated by specialized teams in a separate and independent area of a hospital, known as the Intensive Care Unit (ICU). In these units, various interconnected and interdependent components constitute a complex healthcare system. The ICU system provides patients with the cooperative work of multiprofessional teams, continuous monitoring, and intervention through medicines and medical devices. In this context, we propose studying an ICU from the perspective of the system's natural complexity. Based on expert knowledge, we built a computational model of an ICU. This model considers some relationships that patients establish in the critical care process, as illustrated in Fig. 9. These relationships are modeled as connections between elements of the system, namely patient queue, patient bed, patient equipment, patient staff, and patient adverse event.

The dynamics of the ICU system was simulated using discrete event systems. The entities (patients) wait for resources (beds, equipment, and staff) in a queue. When resources are available, the intensivist evaluates the first patient in the queue based on admission criteria. If admitted, the patient occupies an ICU bed, requiring support from equipment and care from staff (physicians, physiotherapists, nurses, and technicians). During the stay in the ICU, the patient may experience an adverse event, which refers to the harm caused by medical care. In summary, the simulation comprises the following steps: (i) demand for an ICU bed, (ii) ICU bed assignment, (iii) allocation of resources, (iv) patient stay in ICU, (v) patient exit from the ICU (either deceased or discharged alive), and (vi) resources are unallocated. According to the model presented in Fig. 9, each patient waiting for an ICU bed establishes a

<sup>&</sup>lt;sup>1</sup> Systemness is a property in which systems encompass and are encompassed by other systems. However, each system has boundaries that differentiate it from its environment [6].

Table 2

Parameters and respective values defined for sensitivity analysis of the ICU system.

Parameter	Range	Base-value
$n_{beds}$	1–20	10
n <sub>equipment</sub>	15%-200%	100%
$n_{staff}$	15%-200%	105%
rt <sub>ae</sub>	12%-24%	12%
$\mu_{arrival}$	4h-72h	36 <i>h</i>

connection in the queue. The patient admitted to the ICU establishes three connections (one with the bed, one with the equipment, and one with the staff). A patient experiencing an adverse event represents an additional connection in the system.

As shown in Fig. 9, the input parameters analyzed were the number of beds  $n_{beds}$ , the percentage of equipment  $n_{equipment}$ , the percentage of personnel  $n_{staff}$ , the rate of adverse events  $rt_{ae}$ , and the average arrival rate of patients  $\mu_{arrival}$ . The output variable for the sensitivity analysis was the performance of the ICU  $\eta_{icu}$ . By simulating the model, the measure of complexity was calculated using (7). Initially, the optimized case was empirically estimated. The available resources satisfactorily met the demand for ICU services, and the rate of adverse events tended to match the rate of nonpreventable cases. The patient arrival rate was represented by a normal distribution with a mean of 36 h and a standard deviation of 4 h, N(36, 4). Patients who requested ICU beds were classified by priority as follows: 35% Priority 1. 50% Priority 2. 7% Priority 3. 7% Priority 4 and 1% Priority 5. The waiting queue was FIFO (first in, first out), ordered by priority from 1 to 5. The percentage of patients who refused admission to the ICU was 10%.

Regarding resources, the ICU configuration consisted of 10 beds, 100% of the required equipment, and 100% of the workload of the staff, with a margin to add 5% to the workload. This margin accounts for situations where professionals need to extend their workday, such as cardiac resuscitation close to shift changes. The number of resources allocated per patient corresponded to one bed, and between 6% and 12% of the total workload of equipment and staff, following a uniform distribution U(6, 12) for both equipment and staff.

The average length of stay in the ICU (in days) was defined according to the patient's priority, following a normal distribution: N(8, 3) for Priority 1, N(5, 2) for Priority 2, N(7, 1) for Priority 3, N(7, 1) for Priority 4, and N(30, 7) for Priority 5. Based on medical literature, the adverse event rate was established at 12%. For each adverse event that occurred, the patient's stay was extended by 15 to 45 days, represented by a uniform distribution U(15, 45). The ICU mortality rate was established at 20%. Based on this configuration of the computational model, the simulation was performed to: (i) obtain the output value  $\eta_{icu}$  for different scenarios defined by the sensitivity analysis, and (ii) calculate the system's complexity.

This case study established a computational model of the ICU, detailing its structural configuration, resource allocation, and patient flow dynamics. By representing system components and interactions through weighted connections, the model provides a foundation for analyzing how operational parameters influence system performance. This structural framework enables the subsequent application of sensitivity analysis and complexity assessment, supporting a deeper understanding of the adaptive behavior of the ICU under varying workload conditions.

#### 4.2. Applied sensitivity analysis

The sensitivity analysis was conducted using the area method. The input parameters, ranges, and base values are shown in Table 2. The base value of each parameter refers to an empirically defined optimized value considering the desired conditions for ICU operation. Using one-at-a-time measures based on the optimized base case  $\alpha_n^{36h}$ , we plotted the spider diagram shown in Fig. 10.

Based on Fig. 10, we observe that the greatest impact on the performance of the ICU was caused by the negative variation in the number of beds  $n_{beds}$  (in blue color). The percentage of equipment parameters  $n_{equipment}$  (in orange) and the percentage of staff  $n_{staff}$  (in yellow) showed a similar behavior and a significant impact for values below the base value. The adverse event rate  $n_{ae}$  (in purple) varied only from zero to 100%. The base value of the parameter  $n_{ae}$  refers to nonpreventable events, representing the lowest feasible value for the model in question. An increase in the rate of adverse events by 100% led to a performance drop of more than 10%. The variation of the parameter for the average patient arrival rate  $\mu_{arrival}$  (in green) resulted in lower performance values in both analysis contexts (negative variation and positive variation).

Overload and idleness cases were defined based on the optimized value of the average patient arrival rate  $\mu_{arrival} = 36h$ . The overload cases correspond to  $\alpha_u^{12h}$  ( $\mu_{arrival}$  equal to 12h) and  $\alpha_u^{24h}$  ( $\mu_{arrival}$  equal to 24h). The individual cases were defined as  $\alpha_u^{48h}$ ,  $\alpha_u^{60h}$ , and  $\alpha_u^{72h}$ , when  $\mu_{arrival}$  is equal to 48h, 60h, and 72h, respectively. All cases follow a normal distribution with a standard deviation of 4 h. The sensitivity indices were calculated for the different base cases using the area method. These indices are shown in Table 3 and are presented graphically in Fig. 11.

As shown in Fig. 11 and Table 3, the most sensitive parameters are  $n_{beds}$  and  $\mu_{arrival}$ , while the least sensitive parameter is the adverse event rate  $rt_{ae}$ . Since  $rt_{ae}$  only showed a positive variation from the base value, its sensitivity was attenuated. Despite this, the sensitivity indices  $S_{rt_{ae}}$  displayed values close to 10% for both the overload and regularity cases. The sensitivity indices of the parameters under the regularity condition  $\alpha_u^{36h}$  ranged between 10% and 30%. The parameter  $n_{beds}$  became less sensitive as the average patient arrival rate increased, as shown in Fig. 11.

The values of the indices  $S_{n_{beds}}$  and  $S_{rt_{ae}}$  decreased for idle cases compared to the optimized case  $\alpha_u^{36h}$ . In contrast, the sensitivity of the parameter  $\mu_{arrival}$  increased considerably. The parameters  $n_{equipment}$  and  $n_{staff}$  maintained their values between 14% and 21%

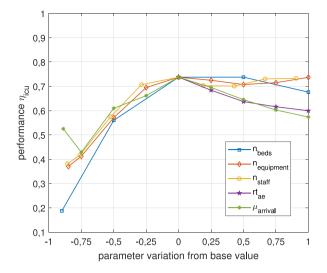


Fig. 10. Spider diagram of the ICU system concerning to optimized base case  $\alpha_u^{36h}$ .

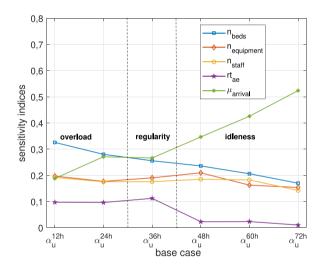


Fig. 11. Sensitivity indices concerning to the base cases in conditions of overload, regularity and idleness.

**Table 3** Sensitivity indices of the ICU parameters related to the output  $\eta_{icu}$  using the area method.

Base case	Condition	$S_{n_{beds}}$	$\mathcal{S}_{n_{equipment}}$	$S_{n_{staff}}$	$S_{rt_{ae}}$	$S_{\mu_{arrival}}$
$\alpha_u^{12h}$	Overload	0.3257	0.1972	0.1919	0.0972	0.1880
$egin{array}{l} lpha_u^{12h} \ lpha_u^{24h} \ lpha_u^{36h} \end{array}$	Overload	0.2802	0.1769	0.1756	0.0963	0.2710
$\alpha_u^{36h}$	Regularity	0.2556	0.1904	0.1757	0.1121	0.2662
$\alpha_u^{48h}$ $\alpha_u^{60h}$	Idleness	0.2359	0.2096	0.1850	0.0228	0.3467
$\alpha_u^{60h}$	Idleness	0.2060	0.1625	0.1824	0.0234	0.4257
$\alpha_u^{72h}$	Idleness	0.1700	0.1537	0.1424	0.0102	0.5237

in all cases analyzed. Under idle conditions, the adverse event had minimal impact on system performance, with  $S_{rt_{ae}}$  indices close to zero. This can be attributed to the availability of resources to assist patients affected by adverse events, who need to remain in the ICU for extended periods.

This sensitivity analysis identified beds  $n_{beds}$  and patient arrival rate  $\mu_{arrival}$  as the most influential parameters that affect ICU performance, with varying impacts in the overload, regularity and idleness scenarios. These results reveal how different operational conditions modulate the relevance of system parameters, providing a nuanced understanding of the adaptive behavior of the ICU.

**Table 4** Probabilities of the connections P(c) of the intensive care unit system concerning to the queue according to the base cases.

Connection c	$\alpha_u^{12h} P(c)$	$\alpha_u^{24h}$ $P(c)$	$lpha_u^{36h} P(c)$	$\alpha_u^{48h}$ $P(c)$	$lpha_u^{60h} P(c)$	$a_u^{72h}$ $P(c)$
Patient priority 1-queue	0.11	0.02	0.06	0.21	0.33	0.34
Patient priority 2-queue	0.68	0.61	0.40	0.47	0.53	0.52
Patient priority 3-queue	0.10	0.16	0.19	0.14	0.07	0.06
Patient priority 4-queue	0.10	0.17	0.29	0.15	0.07	0.07
Patient priority 5-queue	0.01	0.04	0.06	0.03	0.01	0.01

**Table 5** Probabilities of the connections P(c) of the intensive care unit system concerning to the resources and adverse event

Connection c	Probability $P(c)$
Patient-bed	0.10
Patient-equipment	0.09
Patient-staff	0.09
Patient-adverse event	0.12

**Table 6** Complexity measures  $\psi^{icu}(c, \gamma_c)$  for scenarios of overload, regularity, and idleness.

Base case	$\psi^{icu}(c,\gamma_c)$
$\alpha_{u}^{12h}$	153.42
$\alpha_{u}^{24h}$	62.34
$\alpha_{ii}^{36h}$	20.30
$\alpha_{\cdot \cdot}^{48h}$	12.70
$\alpha_{}^{''60h}$	9.63
$\alpha_{u}^{12h}$ $\alpha_{u}^{24h}$ $\alpha_{u}^{24h}$ $\alpha_{u}^{36h}$ $\alpha_{u}^{48h}$ $\alpha_{u}^{60h}$ $\alpha_{u}^{72h}$	7.77

This layer of analysis supports the next stage of the framework, where connection-weighted sensitivity indices will be integrated into the system complexity assessment, allowing a comprehensive evaluation of the operational dynamics of the ICU.

#### 4.3. Complexity analysis

After obtaining the sensitivity indices, the system connections were identified and related to the parameters:  $\mu_{arrival}$  (patient queue connection),  $n_{beds}$  (patient-bed connection),  $n_{equipment}$  (patient-equipment connection),  $n_{staff}$  (patient-staff connection), and  $rt_{ae}$  (patient-adverse event connection). In this modeling, each connection was associated with only one parameter, making the relevance of the connection  $\gamma_c$  consist of the sensitivity index value of the parameter for the different base cases, as shown in Table 3. To apply the complexity metric, we define the values of probability P(c) and connection relevance  $\gamma_c$ . The intensive care unit model, shown in Fig. 9, illustrates the connections that a patient can establish while waiting for a bed or during their stay in the ICU. The connection probabilities in the queue were defined experimentally, considering the priority of each patient, as shown in Table 4. In this context, experimental probability refers to the number of patients per priority in the queue at each instant analyzed, relative to the total number of patients in the queue.

The probability values for resources and adverse events are shown in Table 5. These values were defined according to the model configuration, which includes 10 beds, 100% equipment, 105% workload of the staff, and an adverse event rate of 12%. This configuration remains the same for the base cases  $a_u^{12h}$ ,  $a_u^{24h}$ ,  $a_u^{36h}$ ,  $a_u^{48h}$ ,  $a_u^{60h}$ , and  $a_u^{72h}$ , the difference being the average patient arrival rate. Given that there are 10 beds in the proposed model, the probability that a patient connects to a bed is 0.1, which on average requires 0.09 of resources for equipment and staff. This value of 0.09 refers to the mathematical expectation of the uniform distribution U(6, 12), empirically determined by simulations of the amount of equipment or the workload of staff required by each patient. For adverse events, the connection probability refers to the rate defined for the model, which is 0.12.

The complexity measure  $\psi^{icu}(c, \gamma_c)$  was calculated using (7). The resulting values are provided in Table 6 and illustrated in Fig. 12. As shown in Fig. 12, complexity saturates in scenarios with low resource demand. The value  $\psi_n^{icu}(c, \gamma_c) = 20.3$  marks the onset of this saturation and is therefore interpreted as the natural complexity of the system. It serves as a reference point, approaching the minimum effective level of complexity.

In the simulated ICU scenario, the value  $\psi_n^{icu}(c,\gamma_c)=20.3$  was obtained under a stable, continuous full occupancy condition that lasted 36 h, with no signs of overload or underuse. This configuration reflects an ideal operational state, where the system performance remains consistent and is not affected by abrupt fluctuations in demand or response. Consequently, this value is interpreted as representative of the natural complexity of the system. In contrast to real systems, where natural complexity must be inferred from empirical data and expert knowledge, in simulated systems, this reference can be directly derived from optimized parameters. Thus,  $\psi_n^{icu}(c,\gamma_c)=20.3$  serves as a benchmark for distinguishing between functional regimes and characterizing the dynamic balance between stability and adaptability.

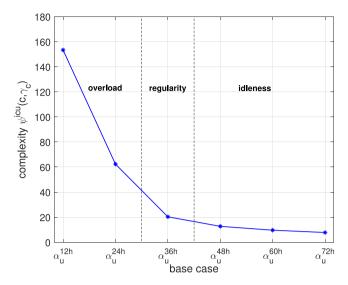


Fig. 12. Complexity based on sensitivity-weighted connections  $\psi^{icu}(c,\gamma_c)$  under overload, regularity, and idle conditions.

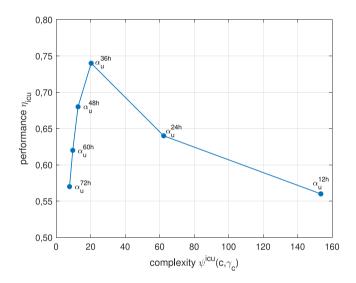


Fig. 13. ICU performance  $\eta_{icu} \times \text{complexity } \psi^{icu}(c, \gamma_c)$  concerning to the overload, regularity and idle scenarios.

#### 4.4. Systems assessment based on natural complexity

The natural complexity refers to the internal and external consistency of the system, considering an adequate workload. Often, systems exceed their physical limits and, in the case of human systems, even psychological limits. Based on these considerations, complexity values higher or lower than natural complexity may indicate system overload or idleness, respectively. In the ICU system, we found that the maximum performance obtained  $\eta_{icu} = 74\%$  corresponds to the scenario with a patient arrival rate of N(36, 4) and an optimized configuration: 10 beds, 100% of equipment, 105% of staff workload, and 12% adverse event rate. The complexity was relatively low for this case  $\alpha_u^{36h}$ . Performance behavior in the complexity measures of the overload, regularity and idle scenarios is shown in Fig. 13.

Taking into account the patient arrival rate of the  $\alpha_u^{36h}$  case as a reference, when the demand for care in the ICU increased, the system was overloaded and when it decreased, the system was idle. According to simulation data, overload was evident due to the capacity of the ICU (more than 90% of the beds used), in addition to the high number of people in the queue (on average, 58 patients in the queue for  $\alpha_u^{24h}$  and 244 for  $\alpha_u^{12h}$ ). In terms of idleness, the percentage of beds in use was less than 70% for the cases  $\alpha_u^{48h}$ ,  $\alpha_u^{60h}$ , and  $\alpha_u^{72h}$ . For patient arrival rates less than 36h, the complexity value increases considerably due to the increase in connections in the queue, and performance decreases: 64% for  $\alpha_u^{24h}$  and 56% for  $\alpha_u^{12h}$ . For patient arrival rates greater than 36h, although complexity values decrease, it can be seen that performance values also decrease: 68% for  $\alpha_u^{48h}$ , 62% for  $\alpha_u^{60h}$ , and 57%

**Table 7**Descriptive statistics of the complexity measures of the samples concerning to the conditions of the ICU system activity.

Statistic	$\psi^{icu}(c,\gamma_c)$	$\psi^{icu}(c,\gamma_c)$	$\psi^{icu}(c,\gamma_c)$
	idleness	regularity	overload
Average value	10.41	21.94	87.91
Standard deviation	1.98	6.42	33.88
Minimum value	7.76	14.21	39.37
Maximum value	13.78	35.74	153.42

for  $\alpha_u^{72h}$ . In these cases, part of the available resources are not used, resulting in maintenance costs. In summary, we have:

$$\begin{cases} complexity \downarrow & performance \downarrow \Rightarrow idleness \\ complexity \downarrow & performance \uparrow \Rightarrow regularity \\ complexity \uparrow & performance \downarrow \Rightarrow overload \end{cases}$$

$$(10)$$

This initial results analysis led us to hypothesize that if the natural complexity represents an adequate level of system activity, then the system is idle when it presents a complexity lower than its natural complexity and is overloaded when it presents a complexity higher than its natural complexity, as given by (11). From the hypothesis test, the differences observed in terms of complexity for the different working conditions were verified. Based on the behavior of the curves shown in Figs. 11 and 12, the overload, regularity, and idleness of the ICU system could be characterized by the patient arrival rate between (i) 12h and 29h, (ii) 30h and 42h, and (iii) 43h and 72h, respectively. We generate samples corresponding to these groups using cubic interpolation, considering the best fit to the curve in Fig. 12. The size of each sample was equal to 60, resulting in 180 complexity values generated in the range between 7.76 and 153.42. The descriptive statistics for each sample are shown in Table 7.

$$\begin{cases} \psi < \psi_n & \Rightarrow idleness \\ \psi > \psi_n & \Rightarrow overload \end{cases}$$
 (11)

Initially, we applied the normality test to the sample data to perform a parametric statistical test. Using the Kolmogorov–Smirnov test [84], the values of  $\psi^{icu}(c,\gamma_c)$  in idle and overload conditions approached a normal distribution since the calculated p-value was equal to 0.165 and 0.200, respectively. Thus, the null hypothesis was maintained that the data distribution is close to normal. The sampling data of  $\psi^{icu}(c,\gamma_c)$  under regularity conditions did not approach a normal distribution since the p-value was equal to 0.037, which is less than 0.05, and therefore the null hypothesis of the normality test was rejected. Based on the results of the normality test, the One-Sample t-Test was performed to compare data in idle or overload conditions with the value of natural complexity  $\psi^{icu}_n(c,\gamma_c)=20.3$ . The null hypothesis was rejected for both cases, with a p-value equal to zero. Taking into account a significance level of 0.05, the observed value t was -38.62 for the idleness case, while the critical t-value was -2.66, and the observed t-value was 15.46 for the overload case, while the critical t-value was 2.66. Therefore, the differences in complexity values under idle or overload conditions are significant compared to the value of natural complexity.

The hypothesis test was also performed using the nonparametric statistical test, the Mann–Whitney test. The comparison of the independent samples of  $\psi^{icu}(c,\gamma_c)$  resulted in a p-value equal to zero for: (i) samples in the condition of regularity and idleness and (ii) samples in the condition of regularity and overload. In both cases, the null hypothesis was rejected, corroborating the alternative hypotheses given by (11). Based on the results of the hypothesis test, we can take the natural complexity  $\psi_n$  as a reference for monitoring systems. This allows us to avoid unwanted situations such as idleness or overload. Although the issues generated by overload are more significant, an idle system also incurs losses due to the cost of maintaining the system's operations. The cost of maintaining an ICU, for example, is considerable. Therefore, it is necessary to aim for a condition of regularity for the system.

This complexity analysis demonstrated that the proposed metric  $\psi^{icu}(c,\gamma_c)$  effectively captures the operational dynamics of the ICU in different workload conditions. By integrating sensitivity-weighted connections and probabilistic interactions, the metric distinguishes between overload, regularity, and idleness scenarios, offering a consistent and interpretable assessment of the state of the system. The validation of natural complexity as a reference threshold further supports its applicability in the monitoring and management of adaptive systems, ensuring that operations remain within optimal performance ranges. This analysis strengthens the methodological foundation for applying the framework to other complex environments.

#### 4.5. Case study: analysis of the real-world startup acceleration ecosystem

This case study is presented to demonstrate the application of natural complexity calculation in a real-world system. The sensitivity of real-world SAE, also known as a business accelerator, was analyzed. The input parameters studied included: entrepreneur profile  $p_e$ , startup profile  $p_s$ , maturity level, and portfolio management  $g_p$ . The maturity level was broken down into two variables: performance  $\eta_s$  and pitch<sup>2</sup> panel score  $p_{ch}$ , obtained at the end of each month. The system output was measured using an evaluation function  $f_{aval}$  defined as the weighted average of the input parameters, according to the analysis context, as shown in Fig. 14(a).

<sup>&</sup>lt;sup>2</sup> A brief presentation lasting three to five minutes aimed at piquing the interest of the other party (investor or customer) in the business, where the opportunity, market, solution, differentiators, and the request are presented [85].

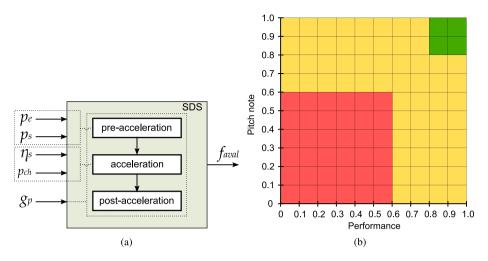


Fig. 14. Context for calculating the sensitivity analysis of the SAE: (a) SAE parameters and (b) management framework for accelerating startups.

Table 8
Parameter data for sensitivity analysis of the SAE

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Parameters	Variation range	Base-value			
$p_e$	0–6	6			
$p_s$	0-32	32			
$\eta_s$	0-1	1			
$p_{ch}$	0-1	1			
$g_p$	0-60	60			

**Table 9** Sensitivity indices of  $p_e, p_s, \eta_s, p_{ch}$ , and  $g_p$  for the SAE.

Method	$S_{p_e}$	$S_{p_s}$	$S_{\eta_s}$	$S_{p_{ch}}$	$S_{g_p}$
Area	0.1935	0.1935	0.1659	0.1659	0.2811
Local conditional variance	0.3844	0.2358	0.0033	0	0.3765

The parameter values were obtained from SAE in Belo Horizonte, Minas Gerais/Brazil [86,87], based on questionnaires and the evolution of SAE in relation to itself and other startups in the accelerator. The maturity level parameter was defined as the square root of the sum of the squares of the performance and pitch score variables, representing the distance between the origin (0,0) and the point characterizing the maturity of each company, according to Fig. 14(b), adapted from Roman [86,87]. The red, yellow, and green areas indicate unsatisfactory, regular, and satisfactory performance, respectively. For the calculation of the evaluation function, the maturity level was moderated by the following weights: 0.15 for the red area, 0.30 for the yellow area, and 0.55 for the green area.

The portfolio management parameter was moderated with weights: 0.70 for Group A, 0.25 for Group B, and 0.05 for Group C. The scores obtained through the portfolio management methodology >48 define Group A,  $36 \le$  Group B  $\le 48$ , and Group C < 36. According to Roman [86,87], SAE with the highest acceleration potential and a high return perspective belong to Group A. Group B includes companies facing development difficulties despite showing initial potential. Group C consists of SAE with a high probability of failure. The evaluation criteria for managing the accelerator's portfolio include team, technology, market, finances, strategy, regulation, and risk. From the data, it was possible to calculate the sensitivity indices using the area and local conditional variance methods. For the application of these methods, the base value of the parameters was defined as the optimized value, as shown in Table 8.

The calculated sensitivity index values are shown in Table 9. The area method visually presents the results in the form of a spider diagram in Fig. 15(a), where  $g_p$  is the most sensitive parameter, and  $p_e$  and  $p_s$  have overlapping curves, as do  $\eta_s$  and  $p_{ch}$ . Using the local conditional variance method, the portfolio management parameter  $g_p$  still had the highest index, greater than 35%. However, the degree of maturity showed values close to zero for both performance  $\eta_s$  and pitch  $p_{ch}$ . The second most sensitive parameter was the entrepreneurial profile  $p_e$ , followed by the entrepreneurial profile  $p_s$ . These results can be explained by the fact that the success of the accelerator depends on the quality of the companies when they enter the acceleration process, observed through the profile of entrepreneurs and the profile of startups, and when they exit, observed through portfolio management.

The SAE was modeled as a set of startup classes, specifically pre-acceleration, acceleration, and postacceleration (or graduated) companies. The connections established by each startup in this system were defined as follows: startup diagnosis for pre-acceleration companies, startup performance evaluation and startup pitch evaluation for acceleration companies, and startup monitoring for

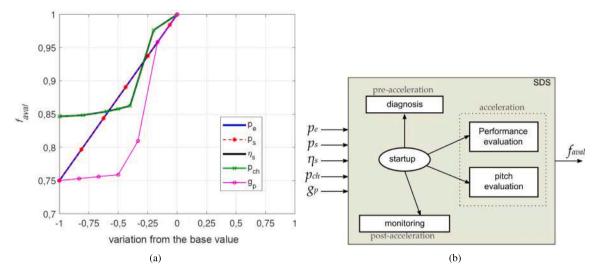


Fig. 15. Context for calculating the natural complexity of the SAE: (a) spider diagram of the parameters and (b) considered connections, input parameters, and output variable used for complexity calculation.

**Table 10** Probabilities of the SAE connections P(c).

Connection c	P(c)
Startup-diagnosis	0.770
Startup-performance evaluation	0.065
Startup-pitch evaluation	0.065
Startup-monitoring	0.100

Relevance  $\gamma_c$  of the SAE connections

Relevance $\gamma_c$
$S_{p_e} + S_{p_s} + S_{g_p}$
$S_{\eta_s} + S_{g_p}$
$S_{p_{ch}} + S_{g_{g}}$
$S_{g_p}$

graduated companies, as shown in Fig. 15(b). To calculate the complexity of SAE, the probabilities of connections P(c) were defined based on the studies by Roman [86,87]. The number of companies in each SAE class was considered in relation to the total number of companies in its portfolio, so the values of P(c) could be calculated, with their values presented in Table 10.

The startups that formed the SAE portfolio were in different stages of development. In this real system, as startups progress through the acceleration process, they require less effort from the accelerator because they gain market presence and acquire autonomy. Therefore, the relevance of the connections was modeled to understand the sensitivity of a greater number of parameters in pre-acceleration connections and fewer in post-acceleration connections. In Table 11, the sensitivity index of the portfolio management parameter  $S_{g_p}$  is present in the relevance of all connections. The indices of the entrepreneur profile  $S_{p_e}$  and the startup profile  $S_{p_s}$  are part of the relevance of the startup-diagnosis connection, while the sensitivity index of the performance parameter  $S_{\eta_c}$  contributes to the relevance of the startup-performance evaluation connection, and the pitch committee score parameter  $S_{\rho_{ch}}$  contributes to the relevance of the startup-pitch evaluation connection.

As the sensitivity indices varied according to the method applied, the relevance values of the connections also fluctuated. Consequently, the complexity  $\psi^{sds}(c,\gamma_c)$  differed for each sensitivity analysis method used. Table 12 shows the complexity of the SAE with 50, 100, and 300 startup. The complexity  $\psi^{sds}(c,\gamma_c)$  increased with the number of startups, as the connections were modeled individually for each startup, disregarding the interconnections among them. The model represents a simplified version of the system and can be adjusted to include the exchange of information, products, and people between SAE companies. In Table 12, the complexity  $\psi^{sds}(c,\gamma_c)$  obtained from the local conditional variance method showed higher values than those obtained from the area method. Among the methods applied, the local conditional variance method is the most representative as it allows analysis across a larger set of feasible points.

Hypothetically, the startup development system could have only one startup. In this case, the following scenarios could occur: the startup being in the pre-acceleration phase, the startup being in the acceleration phase, or the startup being graduated. The results related to the complexity  $\psi^{sds}(c,\gamma_c)$  for these scenarios are presented in Table 13, considering the applied sensitivity analysis

Complexity of the SAE according to the sensitivity analysis method.

$\psi^{sds}(c, \gamma_c)$ 50 startups	$\psi^{sds}(c,\gamma_c)$ 100 startups	$\psi^{sds}(c,\gamma_c)$ 300 startups	Method of sensitivity analysis
52.13	96.29	293.52	Area
63.12	119.22	369.25	Local conditional variance

**Table 13** Complexity  $\psi^{sds}(c, \gamma_c)$  of a single startup in the SAE according to the sensitivity analysis method.

$\psi^{sds}(c,\gamma_c)$ pre-acceleration	$\psi^{sds}(c,\gamma_c)$ acceleration	$\psi^{sds}(c,\gamma_c)$ post-acceleration	Method of sensitivity analysis
0.96	1.41	0.61	Area
1.29	1.27	0.71	Local conditional variance

method. For the local conditional variance method, the highest calculated complexity value corresponds to the pre-acceleration class, followed by the values for the acceleration and post-acceleration classes, respectively. These results are consistent with the observations made in the system, where the accelerator needs to devote considerable effort to the group of companies in acceleration through mentoring, meetings, and pitch evaluations to assess the performance and progress of the companies on a monthly basis.

Regarding the preacceleration class, the accelerator focuses its efforts on preparation and selection events for teams aspiring to form startups. In this phase, entrepreneurs' profiles and business proposals are evaluated, analyzing individual competencies, technical skills, team cohesion, and the team's expertise in relation to technology, market, resources, and business planning. Companies in the postacceleration class require only monitoring from the accelerator, as they typically have recurring revenues. Therefore, any system, whether computational, natural, or human-made, that can map its inputs and outputs, as well as evaluate its connections, can have its natural complexity calculated. All sensitivity analysis methods are used to quantify the influence of parameters on the system. When using sensitivity analysis in the construction of natural complexity, the influence of the parameters is considered. To find the base value, an optimization process or a value indicated by experts is used. In this case, the system is resilient,<sup>3</sup> which means that it is capable of absorbing certain disturbances, adapting to changes, and continuing to function efficiently under adverse conditions.

The natural complexity metric accounts for several key aspects of the system, including the connection structure required for the calculations of sensitivity analysis, as illustrated in Figs. 8, 9, and 15(b). It also considers the parameter configurations required for computing complexity, presented in Tables 2 and 8, and incorporates the workload variations shown in Fig. 12. Together, these components ensure that the metric reflects both the structural configuration and the adaptive dynamics of the system, forming a robust foundation for complexity assessment.

This case study demonstrated the applicability of the proposed natural complexity metric  $\psi^{sds}(c,\gamma_c)$  in a real-world startup acceleration ecosystem, capturing the varying demands across different stages of startup development. By integrating sensitivity-weighted connections and adapting system configurations, the framework produced interpretable complexity measures that reflect the operational effort required for each startup class. The findings confirmed that pre-acceleration startups impose higher complexity, while post-acceleration startups exhibit lower complexity due to reduced dependency on accelerator resources. In addition, the use of multiple sensitivity analysis methods showcased the flexibility of the framework, complementing the insights from the ICU case study and contributing to the generalization of the methodology.

#### 4.6. Comparative validation with conventional complexity metrics

Although conventional metrics are based on different principles, some based on probabilistic properties, such as Shannon entropy, and others on structural or geometric characteristics, such as fractal dimension, degree centrality and clustering coefficient, were all applied dynamically over time in this study. To achieve this, graph structures derived from model sensitivity analysis were used, in which the sensitivity indices of the connections varied at each time point, even though the network's basic topology remained constant. This approach enabled the recalculation of geometric metrics at every time step, reflecting changes in the intensity of interactions among the system components. As a result, originally static metrics were adapted to a dynamic perspective, allowing for consistent temporal comparisons between traditional approaches and the proposed natural complexity metric. This methodology ensures that all metrics, including conventional ones, can capture adaptive variations of the system, although with differing levels of sensitivity.

This methodological adaptation enabled a comparative analysis between the proposed natural complexity metric and traditional approaches, allowing for the assessment of the dynamic behavior of all metrics over time in the case studies. To validate the distinctiveness of the natural complexity metric, a comparison was made with widely recognized metrics in the literature, including Shannon entropy, fractal dimension, algorithmic complexity, degree centrality and clustering coefficient, as presented in Table 1. This comparison was applied to both case studies, the ICU and an SAE, using the same time series data in all metrics. In the ICU case, 30 consecutive days of operation were analyzed, while for the SAE, a 12-month period was considered, reflecting monthly cycles of innovation and development. Fig. 16 displays the temporal evolution of these metrics in their respective case studies.

<sup>&</sup>lt;sup>3</sup> The ability of the system to recover from unexpected events or stresses while maintaining its functions and integrity.

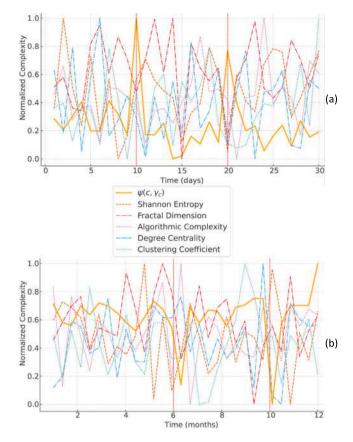


Fig. 16. Temporal evolution of complexity metrics in adaptive systems: (a) ICU and (b) SAE.

The results indicate that, although traditional metrics capture general aspects of system structure or uncertainty, they fail to detect dynamic variations associated with critical operational states during monitoring. In the ICU, for example, the metric  $\psi(c,\gamma_c)$  identified periods of overload, reflected by peaks in the sensitivity of critical connections, while the other metrics remained relatively stable and did not capture such operational changes. Similarly, in SAE, the proposed metric detected instances of systemic idleness, characterized by reduced sensitivity in interactions among actors, which were not captured by conventional approaches. These findings reinforce the ability of the natural complexity metric to provide more interpretable and sensitive diagnostics of adaptive system dynamics, offering a significant methodological contribution compared to traditional complexity metrics.

This methodological advantage is illustrated in Fig. 16, which presents the temporal evolution of complexity metrics in the two case studies. During critical operational periods, such as overload peaks in the ICU (days 10 and 20) and idleness phases in the start-up ecosystem (months 6 and 10), only the metric  $\psi(c,\gamma_c)$  exhibited significant variations, accurately reflecting these adaptive changes. In contrast, traditional metrics remained stable or showed minimal fluctuations, failing to adequately signal such critical states. These results suggest that, when applied to dynamic monitoring, the natural complexity metric offers greater interpretability and sensitivity, enabling more precise and relevant operational diagnostics for the management of adaptive systems.

To ensure the reproducibility of the tests performed, the complexity metrics were implemented according to their respective methodological approaches. Although some conventional metrics provide static assessments of system characteristics, in this study they were applied dynamically over time. For this purpose, sensitivity graph structures derived from simulation models were used for each case study. In these networks, nodes represent system variables or agents, while edges correspond to the interactions between them, weighted by sensitivity indices calculated at each time point. Although the basic topology of the network remains constant, variations in connection weights — resulting from changes in sensitivity indices — allow the continuous updating of geometric metrics, reflecting the adaptive dynamics of the system at each time step.

Shannon entropy was calculated on the basis of the probability distributions of system states, defined from the normalized output variables, such as the aggregated sensitivity of connections within the model. To estimate these distributions, the data were discretized into intervals (bins). The fractal dimension was estimated using the box-counting method, applied to the time series of aggregated system variables, such as sensitivity or interactions among components. Since the sensitivity graphs vary in weighting over time, this variation was incorporated into the fractal dimension calculation, allowing the capture of dynamic nuances within the system. The time series space was discretized across different scales, and the fractal dimension was obtained as the slope of the line in the log-log plot.

The algorithmic complexity was approximated using the compression method, applying Lempel–Ziv compression algorithms to the time series of system variables. Relative complexity was estimated as the ratio between the compressed file size and the original file size. The degree centrality and clustering coefficient metrics were calculated from the sensitivity graphs updated at each time point. The degree centrality was normalized by the maximum possible number of connections, with the global value corresponding to the average centrality of all nodes at each time step. The clustering coefficient was calculated for each node, reflecting the level of interconnectivity among its immediate neighbors, and was updated according to variations in connection weights. The global clustering coefficient was obtained as the average of the local coefficients. All metrics were computed over the aggregated time series based on the sensitivity graphs generated by the simulation models for the ICU and the SAE, allowing dynamic analysis at each time point.

#### 4.7. Interpretative comparison between structural and functional complexity metrics

Although the complexity metric  $\psi(c,\gamma_c)$  can be applied to the same simulated scenarios used to evaluate conventional structural metrics, such as connection density, mean degree and Shannon entropy, it is conceptually inappropriate to perform direct numerical comparisons. Each metric is grounded in distinct theoretical assumptions, analytical purposes, and operational principles, which makes quantitative equivalence between their results methodologically unsound. Instead, we focus on interpretative distinctions with respect to their sensitivity to structural and functional dynamics.

To clarify these limitations, we present a conceptual contrast between  $\psi(c,\gamma_c)$  and conventional complexity metrics. Shannon entropy quantifies distributional uncertainty but ignores interaction intensity and lacks structural context. The fractal dimension captures geometric self-similarity, which is often absent in adaptive systems, and requires explicit spatial or temporal scaling to be meaningful. Algorithmic complexity estimates the length of a minimal description, yet is frequently computationally intractable and disconnected from parameter sensitivity or system output behavior. Degree centrality reflects static network topology but does not account for dynamic variations in connection relevance. Similarly, the clustering coefficient measures local connectivity without considering functional significance or system-level performance.

In contrast,  $\psi(c, \gamma_c)$  is explicitly designed to integrate structural configuration with functional relevance, weighting each connection by its associated parameter sensitivity. Furthermore, it can be computed iteratively at each time step, enabling real-time tracking of adaptive transitions and early detection of inefficiencies. This dual capacity, which captures both the organization and the evolving behavior of the system, separates it from metrics that are purely topological, probabilistic, or descriptive in nature.

#### 5. Discussion

The proposed complexity metric  $\psi(c, \gamma_c)$  integrates several characteristics of the system, such as configuration, arrangement, performance and workload. Using sensitivity indices to weigh the connections, the system is analyzed based on the relationship of its parts to the whole, combining aspects of its internal dynamics (the connections) and external factors (input variables). In addition to the metric  $\psi(c, \gamma_c)$ , this work introduces the concept of natural system complexity, which serves as a fair and reasonable reference for evaluating systems: fair in the sense that the system is effective and reasonable in the sense that the system operates adequately efficiently, without overloading. When using natural complexity as reference, it was observed that the reduction in complexity  $\psi(c, \gamma_c)$  occurred when the resources were used properly or idle, and the increase in complexity  $\psi(c, \gamma_c)$  resulted from system overload.

We also observed that complexity was saturated in scenarios with low resource demand, indicating the cost of keeping the system idle. In the ICU system, the value of  $\psi_n^{lcu}(c,\gamma_c)=20.3$  established the beginning of complexity saturation, leading to the hypothesis that natural complexity represents the minimum complexity of the system in regular activity, neither idle nor overloaded. In systems modeled by entities, resources, and queues, the number of active connections increases according to the system's operating condition: idleness, regularity, or overload, in that order. Therefore, the complexity measure based on weighted connections  $\psi(c,\gamma_c)$  can reflect: (i) configurations with idle resources, (ii) optimized configurations where the system performs well, or (iii) configurations with resource scarcity, where the system capacity does not meet the existing demand. Based on the study conducted by Gomes et al. [22], each configuration can be evaluated by applying the relationship given by:

$$R_c = \frac{performance}{complexity} \tag{12}$$

where  $R_c$  represents the cost in terms of system complexity performance for each unit. When the system is not overloaded, a lower value of  $R_c$  indicates adequate resource availability. In the ICU context, the performance levels in both idle and overload conditions are lower than in normal operating scenarios, as presented in Fig. 13. Therefore, while both extremes are undesirable, overload is particularly concerning. Patients who require intensive care typically have critical conditions that require immediate attention, as their lives can be at risk. Consequently, a significant queue in the ICU suggests a decrease in performance and an increase in system complexity. When queues form and pressure to discharge beds arises, there is a tendency to discharge patients prematurely, which, in turn, increases the likelihood of readmissions from the ICU. The literature suggests that this phenomenon occurs more frequently when the occupancy of the ICU bed exceeds 80%.

The results are consistent with studies characterizing complexity as an emergent property of multiple and interdependent interactions in dynamic systems [4,53]. The inverse relationship between complexity and performance, observed in both overload and idleness scenarios, supports theoretical perspectives that advocate for an optimal operating range governed by the self-regulation of adaptive systems [88,89]. Furthermore, the use of sensitivity indices as weights for internal connections aligns with metric

proposals based on entropy analysis and weighted connectivity, as discussed by Schütz et al. [19] in the context of enterprise architectures, and by MacCormack et al. [90] in complexity cost assessments of organizational systems. This conceptual articulation demonstrates that the natural complexity framework is grounded in established theoretical foundations and contributes a unifying perspective for measuring and managing complex systems, paving the way for a mathematical formulation that encapsulates these principles into an applicable metric.

The proposed complexity metric  $\psi(c, \gamma_c)$  abstracts aspects related to the spatial arrangement of the system, defined by space-time interactions, as an alternative to the metric proposed by Koorehdavoudi [91], which is defined as the product of emergence and self-organization, both derived from the concept of missing information or entropy according to Shannon [40]. The metric  $\psi(c,\gamma_c)$  captures spatial organization through system connections and incorporates entropy to quantify internal uncertainty, which is complemented by external uncertainty derived from sensitivity analysis. Sensitivity analysis is necessary in this context, as even parameters deemed less relevant by uncertainty analysis, due to low variability, can substantially influence model results [12]. Therefore, the complexity metric based on connections weighted by sensitivity indices effectively captures the degree of system complexity as a combination of order and disorder, regularity and randomness, as discussed by Kurths [10].

The  $\psi(c,\gamma_c)$  metric is especially useful for modeling systems with continuous variables and continuous time, filling the gap left by metrics such as fractal dimension, which are not effective in this context due to the lack of geometric patterns in the phase space. In systems such as the ICU, the fractal dimension would also be inappropriate since observations made during simulation are abstractions of the system in operation and would likely not exhibit self-similarity. Therefore, the proposed metric quantifies complexity related to the system's dynamics across various scenarios, considering both the difficulty in describing the system and the degree of its organization. The system's configuration, modeled in terms of connections, is used to describe it, while the relevance of the connections, determined by sensitivity measures, defines the degree of organization. These measures can be obtained using local or global methods. In the proposed study, local sensitivity analysis is used. However, it is important to note that the impact of non-linearity in the system's operating range was not addressed. To address this, global sensitivity analysis should be considered to expand the understanding of the natural complexity of the system  $\psi_n(c,\gamma_c)$ . Despite this, the area method proved to be suitable for identifying more or less sensitive parameters, even in contexts with a limited number of scenarios for analysis.

Given the variety of available sensitivity analysis methods, along with the proposed approach, it is recommended to conduct research to determine which method is most suitable for integrating the complexity calculation methodology  $\psi(c, \gamma_c)$ . When choosing to apply global methods, it is important to consider that the time spent simulating systems and computational costs can be considerable compared to local analysis. However, local analysis may not be the best choice if the initial parameter values are in the instability region of the system. In this case, the global sensitivity analysis may be more appropriate, as it covers both the instability and stability regions of the system, considering the impact of non-linearity throughout the operating range of the system [92]. This analysis extends to determining the natural complexity of the system,  $\psi_n(c, \gamma_c)$ , which can be calculated locally or globally, depending on the sensitivity analysis method adopted. When natural complexity is calculated locally, it is advisable to conduct a robustness study to assess the system's sensitivity to parameter changes [93]. For example, in the ICU system, the parameter sensitivity indices varied between 11 and 25 when the system operated at its natural complexity. These values indicate that all parameters are relatively sensitive in a balanced manner. In this situation, it is fundamental to assess whether the system's configuration is robust and consider alternative configurations close to the optimized point, to define the resilience region, which would also correspond to the natural complexity region of the system.

Although the complexity metric  $\psi(c,\gamma_c)$  is just one of several available, the concept of natural complexity in systems stands out as a promising subject for future research. The proposed methodology, along with the use of  $\psi_n(c,\gamma_c)$  as a reference point, proved to be effective in open discrete event systems, where the queue size can indicate the system's workload. It is important to validate this hypothesis in other types of model, such as those with continuous variables and continuous time, continuous variables and discrete time, or discrete variables and discrete time. Although measuring complexity can be a challenge similar to that faced in describing electromagnetism before Maxwell's equations [68], the studies conducted can be unified through the development of a common methodology. The idea of natural complexity, for example, can be applied using different system complexity metrics. In scenarios like the COVID-19 pandemic, intensive care represents a significant challenge. The critical condition of patients requires immediate care, and high demand can compromise performance, resulting in a higher mortality rate and increased complexity of the healthcare system [94]. During the pandemic, the ICUs experienced overload, with a high demand for beds, equipment, and important supplies. This led to extensive efforts by healthcare professionals, which affected their physical and mental health [95]. Strengthening the response of the healthcare system by creating emergency capacity represents an attempt to restore the natural complexity of the system when resources are adequate to meet demand.

The proposed complexity metric  $\psi(c,\gamma_c)$  captures both order and randomness employing probabilities in (7). The connections in the ICU model are defined based on the real dynamics of the system, reflecting its practical application [96]. Sensitivity analysis is performed locally around the values of optimized parameters, although global sensitivity analysis can also be performed to improve the precision of natural complexity assessment, since  $\psi(c,\gamma_c)$  depends directly on these values. The ICU case study evaluates complexity under predefined conditions; however, the natural complexity metric  $\psi(c,\gamma_c)$  can be used to improve system design or operation under varying conditions, facilitating decision-making. The analysis of results indicates that the natural complexity metric provides valuable insights into the system's behavior under different workload conditions. The significant variation in complexity between overload, regularity, and idleness scenarios demonstrates the metric's sensitivity to changes in the system's operational environment. Implications for system management include monitoring and decision-making, where the complexity metric serves as a continuous monitoring tool, allowing managers to quickly identify situations of overload or idleness. This early identification facilitates the rapid implementation of corrective measures, thus improving the system's response to adverse conditions.

These findings gain practical relevance when applied to real-world scenarios. Comparative analysis demonstrated that the natural complexity metric  $\psi(c,\gamma_c)$  not only exhibits greater sensitivity to adaptive dynamics of systems, but also provides more interpretable operational diagnostics, allowing concrete applications in different contexts. To illustrate this potential, in the ICU, the ability to detect overload conditions, associated with increased sensitivity of critical connections, represents a significant advantage for the efficient management of resources such as beds, equipment, and medical teams. By capturing internal system variations in real time, the proposed metric supports decision making for the appropriate redistribution of these resources, enhancing patient safety and care efficiency, both necessary in high-complexity environments such as public healthcare.

Although the benefits are evident in healthcare care, the metric also proves to be effective in other domains. Similarly, in SAE, identifying periods of systemic idleness using the metric  $\psi(c,\gamma_c)$  allows the optimization of innovation cycles, improving the productivity and sustainability of the participating companies. In this context, the early detection of declines in the sensitivity of interactions among system actors provides valuable information to reconfigure strategies, intensify mentoring activities, or support fundraising efforts, contributing to the strengthening of the innovation ecosystem. Unlike conventional metrics, which provide static assessments of systemic characteristics, the proposed natural complexity framework introduces a dynamic reference level, enabling continuous monitoring of system operational states and the identification of suboptimal conditions such as overload and idleness. This operational granularity reinforces the practical applicability of the proposed metric, particularly in scenarios where systemic performance directly influences social and economic outcomes.

In resource allocation, complexity analysis can guide the efficient allocation of human and material resources. In high-complexity situations, task redistribution or a temporary increase in personnel may be necessary to maintain the quality of care. In strategic planning, understanding the complexity of the system in different scenarios allows for a more resilient analysis. Managers can simulate various scenarios and prepare contingency plans specific to high-complexity situations. In addition, the complexity metric can be integrated into continuous improvement processes, providing quantifiable data to assess the impact of new policies or interventions on the system, facilitating evidence-based improvements. The proposed method for natural complexity can be utilized as presented in the real-world case study of the SAE. Furthermore, it can be applied to any industrial system that uses sensors for data collection. Environments that have implemented Industry 4.0, for example, can collect data directly from their sensors and continuously use them in the proposed method. The collected data will not only indicate problems in the system, but will also allow the application of artificial intelligence to the input data values and natural complexity, enabling preventive analyses.

However, several limitations of the current approach should be acknowledged. Despite the applicability demonstrated in the case studies, the methodology presents constraints that must be explicitly addressed. First, it relies predominantly on local sensitivity analysis, which may limit its applicability in systems characterized by significant nonlinearities or multiple operational equilibria. Furthermore, the approach depends on the availability of a representative and well-calibrated model to define internal connections and critical parameters, potentially restricting its use in systems where modeling is incomplete, uncertain, or subjective. Another limitation concerns the lack of consideration for higher-order interactions among input parameters, which could be addressed through second-order global sensitivity analyses.

Future research should focus on developing extended versions of the complexity metric that incorporate global sensitivity approaches, along with empirical validations in diverse domains to assess the generalizability of the framework. These initiatives would improve the reliability of the proposed metric and strengthen its potential as a decision support tool.

In future applications, the natural complexity threshold can also require recalibration depending on the operational context of the system, including load balancing strategies or evolving configurations. This recalibration would allow the framework to maintain diagnostic precision in systems subject to variable demand patterns or dynamic resource allocation, ensuring that complexity assessments remain aligned with operational realities. Such environments exemplify adaptive systems, where continuous re-calibration of complexity thresholds can enhance operational resilience, allowing systems to dynamically adjust to changing workloads, resource availability, or external demands. This adaptability is important for maintaining optimal performance in industry 4.0 ecosystems.

#### 6. Conclusion

In our study, we introduced the concept of natural complexity of systems and developed metrics to analyze both sensitivity and complexity. We modeled the system in terms of its connections and analyzed it considering its interactions with the environment, variations in input parameters, and their effects on output variables. Using sensitivity indices, we assessed the importance of each connection. In summary, sensitivity analysis allowed us to: (i) quantify the influence of parameters, (ii) understand the relationships between input and output parameters, and (iii) evaluate the resilience of the optimized solution. Furthermore, resilience studies can help define the range of natural complexity for the system, ensuring its effective and efficient performance even in the face of possible changes.

The complexity metric based on weighted connections,  $\psi(c,\gamma_c)$ , addressed several aspects of the system: connection structure, adjusted parameter configuration, performance used as output to determine connection relevance, and workload evaluated by the total number of connections. Considering the overall workload of the system, we confirm the hypothesis that the system would be underutilized or overloaded if the complexity diverged from the natural complexity  $\psi_n(c,\gamma_c)$ . Complexities lower than  $\psi_n(c,\gamma_c)$  indicated under-utilization, while higher complexities suggested system overload. We also observed that natural complexity represents the minimum level required to maintain regular system operation with high performance, as indicated by the saturation of complexity.

The concept of natural complexity applies to both natural and human-made systems, optimizing performance in balance with the nature of the system. The complexity metric  $\psi(c,\gamma_c)$  indicates that natural complexity  $\psi_n$  can serve as a valuable reference point for system analysis and monitoring. In systems, one of the primary goals is to avoid both idleness and overload. Applying the metric  $\psi(c,\gamma_c)$  requires modeling the system in terms of connections and evaluating the resulting impact of variations in input parameters. From this analysis, the sensitivity indices necessary to weight the connections are derived. Furthermore, it is important to highlight that the metric  $\psi(c,\gamma_c)$  can be implemented in experiments with real systems, thus avoiding the need for computational model development.

The proposed natural complexity framework not only enhances interpretability, but also establishes a robust foundation for adaptive system management, bridging structural and dynamic perspectives within a unified quantitative approach. Future studies may extend this approach to other domains, such as industrial automation systems, smart energy grids, and distributed system performance analysis, further establishing natural complexity as a robust method for analyzing and supporting decision-making across multidisciplinary complex IS/IT-based systems.

#### CRediT authorship contribution statement

Viviane M. Gomes Pacheco: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. Gabriel A. Wainer: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. Flavio A. Gomes: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. Weber Martins: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Validation, Software, Resources, Methodology, Investigation, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. Marcella Scoczynski R. Martins: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. Clóves Gonçalves Rodrigues: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Validation, Software, Resources, Methodology, Investigation, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. Wesley Pacheco Calixto: Writing – review & editing, Writing – original draft, Visualization, Formal analysis, Data curation, Conceptualization. Wesley Pacheco Calixto: Writing – review & editing, Writing – original draft, Visualization, Formal analysis, Data curation, Conceptualization.

# Declaration of competing interest

The author declare that we have no financial interests or conflicts of interest that could potentially influence the objectivity, integrity, or impartiality of our research findings. Specifically:

- 1. Financial Support: The research conducted and the preparation of this manuscript received no external financial support, grants, or funding from any public or private entity.
- 2. Patents: We confirm that there are no patents associated with the research work presented in this manuscript, and no patent applications have been submitted during the course of this study.
- 3. Salary Reimbursement: The authors involved in this research project have not received any salary, fees, or reimbursements related to the publication of this work. The research was conducted as part of our academic and professional activities, and no financial compensation has been sought or received.

The authors hereby affirm that this Declaration of Interest accurately reflects our financial and non-financial relationships, and we acknowledge our responsibility to promptly inform the editorial board of any changes in our circumstances that may impact this declaration during the review process.

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# Data availability

Data will be made available on request.

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