Analytical method for calculating the sensitivity index of system parameters

Viviane M. Gomes
NExT Research Group
Federal Institute of Goias
Goiania, Brazil
viviane.gomes@ifg.edu.br

Alfredo O. Assis
Eletrical, Mechanical &
Computer Engineering School
Federal University of Goias
Goiania, Brazil

Calebe A. Matias
NExT Research Group
Federal Institute of Goias
Goiania, Brazil

Julyana P. Saraiva
NExT Research Group
Federal Institute of Goias
Goiania, Brazil

Flávio A. Gomes
Eletrical, Mechanical &
Computer Engineering School
Federal University of Goias
Goiania, Brazil

Gabriel A. Wainer
Carleton University Centre for
Visualization and Simulation
Carleton University
Ottawa, Canada

Beatriz S. Lima
NExT Research Group
Federal Institute of Goias
Goiania, Brazil

Alana S. Magalhães
NExT Research Group
Federal Institute of Goias
Goiania, Brazil

Wesley P. Calixto
NExT Research Group
Federal Institute of Goias
Goiania, Brazil
wpcalixto@pq.cnpq.com

Paulo H. R. Flores
NExT Research Group
Federal Institute of Goias
Goiania, Brazil

Junio S. Bulhões
NExT Research Group
Federal Institute of Goias
Goiania, Brazil

***Abstract*—This work proposes an analytical method for calculating sensitivity index. The proposed method is based on one-at-a-time measurements and calculates the difference between output values and the base solution to define the impact caused by each parameter in the system. The results indicated that the analytical method provides adequate values for the sensitivity index even when the parameters have minor variation in relation to the base value, presenting coherent response with graphical analysis. The proposed method satisfies a wide range of systems and assigns value to the influence of the parameters, contributing to the decision-making process, calculation of system complexity, identification of systems, among other situations.**

***Keywords*—sensitivity index, system parameters, analytical method.**

I. Introduction

Systems are often analyzed through the relationship between inputs and outputs. From the simulations or experiments, this relationship can be studied in order to define the sensitivity of the parameters, which allows to indicate: i) the most relevant parameter for analysis, in order to reduce the uncertainty of the output, ii) the least influential parameters, iii) the parameters that make the output more susceptible to changes, iv) the parameters of higher correlation with the output, and v) the consequences of changing the parameter values when the system is running [1].

The sensitivity analysis can be understood as the quest to quantify the relative contribution of each input to the output of the model [2], indicating as the output depends on the input parameters [3]. Commonly, the sensitivity analysis is accompanied by the uncertainty analysis [4], [5]. The essential difference between them is that the uncertainty analysis evaluates how uncertain a given conclusion is and the sensitivity analysis indicates where this uncertainty comes from [6].

In the study of systems, we search for the model that represents the system properly, preserving its essential characteristics. The size of the model should not be confused with its complexity. In this case, the sensitivity analysis can contribute to identify the complexity, verifying if the model is processing the parameters in the way that it should, and to moderate the complexity by reducing the less relevant input variables, making the computational effort lower during the operation of the model [7], [8].

Although recent techniques are more common, work involving sensitivity analysis dates back a few decades. Charnes [9] used sensitivity analysis in the hypothetical logistics model of the oil industry. Schumann [10] explained the use of sensitivity

Thanks for CAPES, CNPq and FAPEG.

analysis to find the most appropriate measurement scale in practical situations, especially in those where subjective evaluation of samples is required. Golub [11] proposed analysis that aims to address practical problems in which continuous variables can not be measured in this way.

Sensitivity analysis methods are usually divided into: i) local and ii) global. Local analysis consists of one-at-a-time measures, where one parameter is varied while the others are kept fixed. Global analysis is performed by varying all parameters at the same time within finite (or even infinite) region [3]. Another frequent classification is the analysis of sensitivity in mathematical, statistical or graphic form [7], [12].

Some authors have devoted themselves to reviewing and improving methods of sensitivity analysis found in the literature [1], [7]. Reitsma [13] discusses the advantage of bivariate analysis over the standard method of meta-analysis and both evaluate the sensitivity and specificity of systems. Allaire [14] uses sensitivity analysis to perform structural optimization. Homma [2] works with higher-order global sensitivity index for non-linear models. Iman [4] investigates the applicability of uncertainty and sensitivity analysis techniques.

According to [3], a new method for calculation of global sensitivity index is presented, based on the Fourier amplitude sensitivity test, indicating the quantitative index of the contribution of each input parameter to the output. Van [15] uses one-at-a-time method combined with latin-hypercube parameter space sampling to obtain global sensitivity analysis for several parameters with less computational effort.

Several studies focus on the practical application of sensitivity analysis. In consonance with [16], there is an evaluation of the relevance of the uncertainties of the emission rate of chemical components associated to the potential range of certain coefficients values. Zhang [17] verifies which factors have greater influence on the economic viability of biodiesel production. By means of sensitivity analysis, Thompson [18] investigates the improvements of real-time weather forecasts on the icing formed in aircraft because of water in the clouds. In [5], there is notable need for sensitivity analysis in biological systems due to their high level of uncertainty.

Sensitivity analysis is also used to make robust models. Robust model is insensitive to variation in parameters, keeping output practically constant [19] [20]. According with [21] there is a sensitivity analysis of results obtained in other studies that relate population expansion with economic indicators in order to define them as robust or sensitive to small variations. Hegre [22] manipulates model with 88 parameters to illustrate civil war in literature and uses sensitivity analysis to evaluate the empirical results available on the subject, since most of them are not robust due to the divergence of topic definitions and periods.

To analyze sensitivity from one-at-a-time measures on a system with k parameters, $k-1$ parameters must be setted in their base value and vary only one parameter at a time. The set of base values is known as base case, which corresponds to the best bet for the parameters or the optimal solution. The system response to this base case is the base solution, graphically represented by the point of intersection of the sensitivity curves. The graph formed by the sensitivity curves is called spider graph [23].

Parameters shall have a specific range, which defines the set of viable values that each of them can assume. In practical situations, setting the parameter range means setting the operating range of each one to keep the system running. Due to operational or optimization issues, the parameters may have constraints on the range definition and can present a reduced range.

Considering that one-at-a-time measures may adequately represent the behavior of certain classes of systems, the purpose of this work is to present generalization of methods developed by [24] to calculate sensitivity index. Section II sets out the proposed analytical method. Comparisons are made, in Section III, between the analytical method and the method of the area proposed by [24].

II. Methodology

The methodology of this work proposes an analytical method developed from the work of [24], in order to solve the problem of distinct intervals of variation from the base values. To evaluate the influence of each parameter on the system response, one-at-a-time measures are used, where the values of one parameter are varied while the other parameters are maintained at their base value. For k input variables, there is a system output $y = f(x_1, x_2, \cdots, x_k)$ and $\beta = y$ when y is the base solution.

The proposed method is defined by the following steps.

1) obtain system output values according to one-at-a-time measures;
2) perform linear transformation of normalization:
   a) define difference between the output values and the base solution;
   b) calculate the value resulting from the impact of each parameter on the system;
   c) normalize the resulting values to set the sensitivity index per parameter.

The output of the system is composed by components of the input variables. Thus the difference between the output values and the base solution β results in the impact of the parameter that has been varied since the components of the other parameters kept constant are canceled considering they are the same as the base solution. After isolating the impact by parameter, the resulting impact of each input variable is calculated.

Linear transformation removes trends in cases of distinct amounts of one-at-a-time measures for the parameters. Another solution is to treat the data before applying the method by reducing or increasing the size of the measurement samples per parameter so that they all have the same size. Therefore, the smallest or largest sample size is used as a reference to treat the others. In case of reduction of samples, it is suggested that the measures with output value less impacting should be removed. As for the increase of the samples size, one can perform interpolation from the available measurements for each parameter.

The proposed method, represented by (1), is the generalization of the method of sum of differences presented by [24], which considers sample points in the spider graph to perform analysis of the impact of each parameter. The proposed method dismiss the graphical analysis, requiring only the one-at-a-time measures.

$$S(x_i) = \frac{\frac{1}{n} \cdot \sum_{j=1}^{n} |y_{ij} - \beta|}{\sum_{i=1}^{k} \left(\frac{1}{n} \cdot \sum_{j=1}^{n} |y_{ij} - \beta| \right)} \tag{1}$$

where i is the index of the parameter, n is the number of one-at-a-time measures of the parameter x_i, y_{ij} is the output of the system for j-th measure of x_i, k corresponds to the number of parameters and β is the base solution.

III. Results

Simulations and experiments with different ranges of variation were performed for the parameters, due to the constraints of each variable, e.g. to maintain values in the range of operation as an electric generator. The proposed analytical method was compared to the method of the area, developed by [24], which establishes the base axis for the value corresponding to the base solution β, thus forming a line parallel to the abscissa axis in the spider graph. In the method of the area, the sensitivity index $S_a(x_i)$ consists of the relation between i) the area of the polygons formed by the base axis and the curve of the parameter x_i and ii) the total area.

The case studies 1 and 2 comprise the sensitivity analysis performed from simulated data and the case study 3, from experimental data. In the first case study, the sensitivity of the variables x_1 and x_2, which have the same range of variation in the spider graph, was verified. In the second case study, the sensitivity index of the variables x_3 and x_4 was calculated, and the range of x_3 corresponds to only 25% of the range of x_4. In the third case study, electrical system parameters were studied, which have variation restrictions.

A. *Case Study 1: Simulated data with parameters ranging from -100% to 100% from the base value*

From the simulations, were performed one-at-a-time measures of the parameters x_1 and x_2 for output y, shown in the spider chart in Fig. 1. These measures were based on the base value and on the range of the parameters, as presented in Table I, allowing the variation of x_1 and x_2 in the range of -100% to 100% of the base value.

As observed in the spider graph in Fig. 1, the parameter x_2 (in red) is more sensitive than x_1 (in blue), because it produces greater impact on output y. This observation is confirmed by the values of the indices presented in Table II, where x_1 has a sensitivity of approximately 41% and x_2, 59%.

TABLE I
BASE VALUES AND RANGE OF THE PARAMETERS x_1 AND x_2.

Parameter	Base value	Range
x_1	10	[0 30]
x_2	50	[0 100]

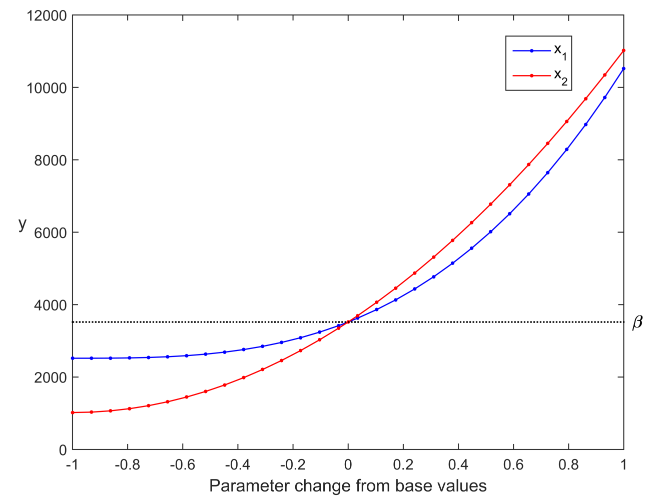

Fig. 1. Spider graph of simulated parameters with the same range of variation from base values.

Sensitivity indices (S_a and S) were calculated using methods of the area and analytical , respectively, resulting in the values given in Table II. Note that indices values of $S_a(x_1)$ and $S(x_1)$ are very close, as well as the values of $S_a(x_2)$ and $S(x_2)$. This result indicates that methods are equivalent when parameters have the same range of variation from the base value or in the same ratio.

TABLE II
SENSITIVITY INDICES OF THE PARAMETERS x_1 AND x_2.

Output	$S_a(x_1)$	$S_a(x_2)$	$S(x_1)$	$S(x_2)$
y	41.18%	58.82%	41.41%	58.59%

B. *Case Study 2: Simulated data with different variations of the parameters from the base value*

Parameters x_3 and x_4 have been varied one at a time to measure the output y through simulations, where measurements are arranged in the spider graph in Fig. 2. These measures were based on the base value and ron the range of the parameters presented in Table III. Although the base values of x_3 and x_4 are equal to x_1 and x_2, parameters x_3 and x_4 impact the system differently, in addition to having different ranges.

TABLE III
BASE VALUES AND RANGE OF THE PARAMETERS x_3 AND x_4.

Parameter	Base value	Range
x_3	10	[8 13]
x_4	50	[0 150]

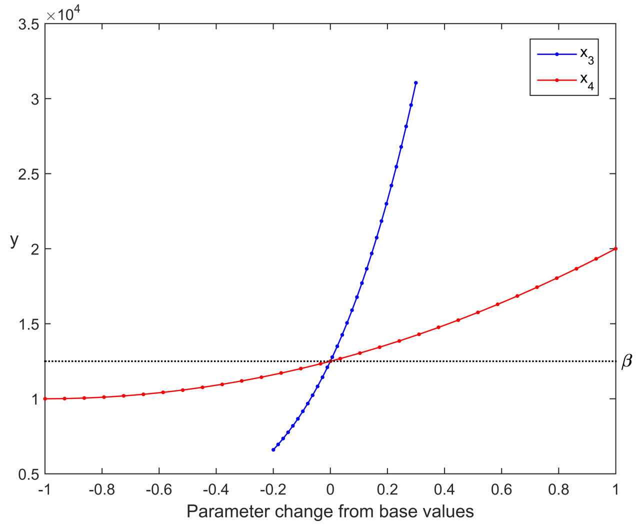

Fig. 2. Spider graph of the simulated data for parameters with different ranges of variation from the base values.

The sensitivity indices (S_a and S) corresponding to the application of methods of the area and analytical, respectively, for variations of x_3 and x_4 are shown in Table IV . It is observed in the graph in Fig. 2 that the input variable x_3 (in blue) has a high impact on the system, even for a limited variation of -20% to 30% from its base value. While the method of the area asserts that $S_a(x_3) = 38.13\%$, the analytical method defines that $S(x_3) = 71.13\%$, being consistent with the graphical analysis.

TABLE IV
SENSITIVITY INDICES OF THE PARAMETERS x_3 AND x_4.

Output	$S_a(x_3)$	$S_a(x_4)$	$S(x_3)$	$S(x_4)$
y	38.13%	61.87%	71.13%	28.87%

C. Case Study 3: Experimental data with different parameter variations from base value

In this case study, was used experimental data developed by [25] on the repowering of the electric system. The analyzed system is composed by a synchronous generator that operates in parallel with induction generator in the common bus subjected to non-linear load. Through the sensitivity analysis, the objective is to present the relation of the influence between inputs and outputs of the repowering system.

Laboratory tests were performed for the Electrical Interconnected Power System indicated in Fig. 3, where M_1, M_2, M_3, and M_4 are the measurement points. Generating units, synchronous and induction, are connected in parallel in order to feed the nonlinear load N_L, consisting of triac rectifier that feeds sets of lamps. Two phases are $5kW$ and the third phase is $4kW$. In order to properly regulate rotations of the generator, S_G and I_G, were used diesel engine and induction motor with frequency inverter, respectively.

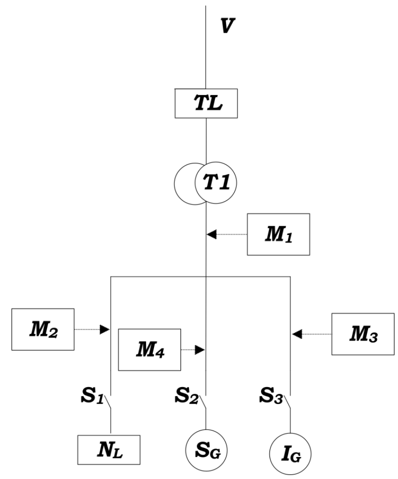

Fig. 3. Interconnected Power System.

Components and values of the Interconnected Power System in Fig. 3 are shown in Table V.

TABLE V
COMPONENTS AND VALUES OF A ELECTRICAL INTERCONNECTED POWER SYSTEM.

Variable	Component	Value of Used Components
S_G	Synchronous Generator	37kVA, 380V
	(main generator)	three-phase, salient poles
		4 poles, 60Hz
I_G	Induction Generator	7.5kVA, 380V
		three-phase, cage rotor
		4poles, 60Hz
T_L	Primary Feeder	three-phase, 13800V, 60Hz
T_1	Transformer	750kVA, 13800/(380/220)V,
		grounded δ/Y
N_L	Nonlinear Load	14kW three-phase, 380V, 60Hz
S_1, S_2, S_3	Switch	

Sensitivity analysis of the system is performed through the data collected in the meter M_1, for each variation of the input parameters. The methodology used to collect the data using the bench is described in the following steps: i) turn on switchs S_1, S_2 and S_3 and ii) vary one parameter at a time, while the others are fixed at their base value . The varied parameters were: mechanical power of the primary machine of the synchronous generator, by means of the current I_{SG}, excitation voltage of the synchronous generator field V_f, induction generator velocity ω_{IG} and the nonlinear load rectifier firing angle θ.

For this study, the output variables measured in M_1 were: i) active power and ii) reactive power. Parameters base values, their respective ranges and percentages of variation in relation to the base values are presented in Table VI. The data collected experimentally are shown graphically in Fig. 4 and Fig. 5, corresponding to the one-at-a-time measures for active power P, in Watt (W), and *reactive power* Q, in Volt-Ampere reactive (VAr), respectively.

TABLE VI
BASE VALUES AND PARAMETER CONSTRAINTS EXPERIMENTALLY VARIED.

Parameter	Base value	Range	Variation from base value
I_{SG}	$36.03A$	[21.91 36.03]	$[-39.19\%\ \ 0\%]$
V_f	$41.5V$	[38.8 48]	$[-6.51\%\ \ 15.66\%]$
ω_{IG}	$1840rpm$	[1815 1855]	$[-1.36\%\ \ 0.81\%]$
θ	132.83^o	[80.85 140.5]	$[-39.13\%\ \ 5.77\%]$

It is observed that the current of the synchronous generator I_{SG} is the most sensitive parameter, with the greatest impact on the two outputs. This observation was confirmed by the calculated sensitivity indices, which presented values above 50% for both methods and outputs, arranged in Table VII and Table VIII.

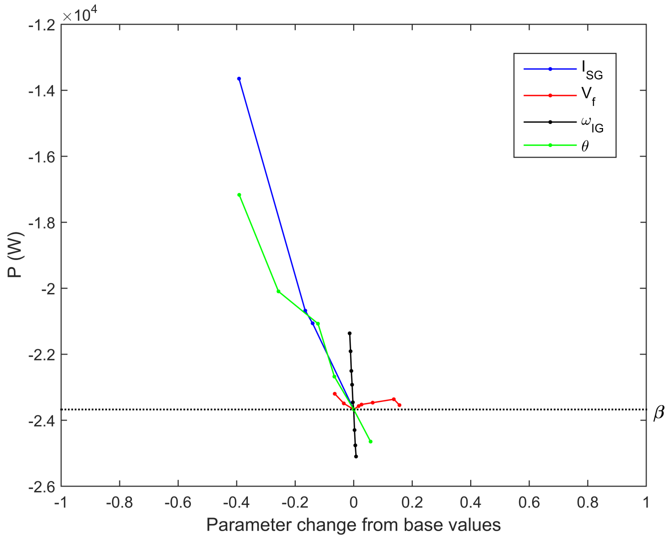

Fig. 4. Spider graph of experimental active power data from power generation system.

Analyzing Fig. 4 and Fig. 5, it is observed that the excitation voltage of the synchronous generator has higher sensitivity in the reactive power output. In fact, when it is desired to change the reactivity of a synchronous machine, the excitation of the machine is changed to vary the field thereof. The synchronous machine can provide or supply reactive power to the network.

Table VII and Table VIII display the sensitivity indices S_a and S for the parameters I_{SG}, V_f, ω_{IG} and θ calculated by the method of the area and analytical method, respectively. By analyzing the values of the indices and the spider graphs, the analytical method stands out by the adequacy of the values of the indices to the generated impact, as observed in the speed index of the induction generator, $S(\omega_{IG}) = 12.26\%$. By the method of the area, this parameter obtained index $S_a(\omega_{IG}) = 0.69\%$, but the parameter ω_{IG} had a considerable impact even at values close to its base value, included in its small range of variation, $[-1.36\%\ \ 0.81\%]$.

In fact, in order to change the active power of the system is necessary to: i) increase the power generated by the synchronous generator, ii) increase the power generated by the induction generator, and iii) change the load consumption, confirming the data presented in Table VIII for I_{SG} of 54.64%, ω_{IG} of 12.26% and θ of 30.75%. To change the reactive power output of the system the greatest impact is given by parameters I_{SG} of 63.81% and V_f of 28.20%, as Table VIII, confirming that the synchronous generator is the most responsible for the system reactive variation.

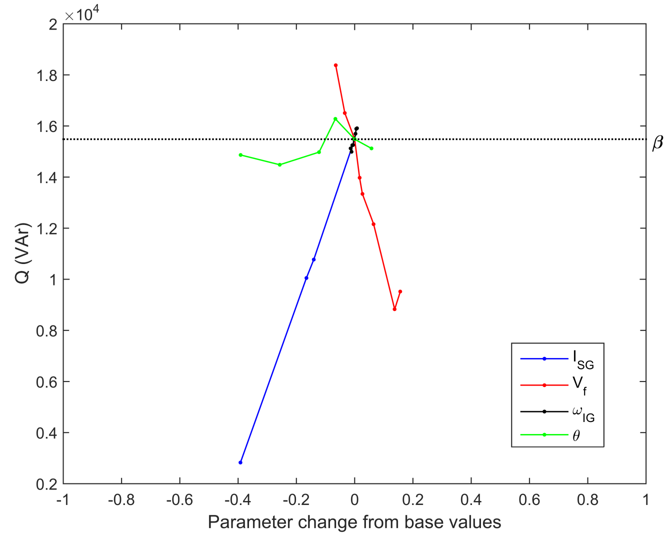

Fig. 5. Spider graph of experimental reactive power data from power generation system.

TABLE VII
SENSITIVITY INDICES CALCULATED BY THE METHOD OF THE AREA FOR EXPERIMENTAL DATA.

Output	$S_a(I_{SG})$	$S_a(V_f)$	$S_a(\omega_{IG})$	$S_a(\theta)$
P	55.57%	1.56%	0.69%	42.18%
Q	71.14%	20.32%	0.15%	8.39%

TABLE VIII
SENSITIVITY INDICES CALCULATED BY THE ANALYTICAL METHOD FOR EXPERIMENTAL DATA.

Output	$S(I_{SG})$	$S(V_f)$	$S(\omega_{IG})$	$S(\theta)$
P	54.64%	2.35%	12.26%	30.75%
Q	63.81%	28.20%	2.48%	5.51%

Sensitivity indices calculated according to the analytical method adequately express the electrical system under study. Even with the restrictions of variation of the parameters in relation to their base value, the analytical method presented values of sensitivity appropriate for all the parameters.

IV. CONCLUSION

This work proposed an analytical method based on one-at-a-time measures for the calculation of sensitivity index. Considering results obtained by the method of the area, continuous approach of the method of sum of differences, it was verified that the evolution in time represented by the behavior of the curves of the spider graph and used in the calculation of sensitivity can cause inconsistencies and errors, being necessary only measurements that express the impact of the parameters in the system under analysis. The proposed analytical method adequately defines the most sensitive parameters, regardless of existing variation constraints, satisfying a wide range of systems. By assigning value to the influence of parameters, this method contributes to improve processes in decision making, calculation of system complexity, identification of systems, among other areas.

Acknowledgment

The authors would like to thank Coordination for the Improvement of Higher Education Personnel (CAPES), the National Counsel of Technological and Science Development (CNPq) and Research Support Foundation of Goias State (FAPEG) for financial support research and scholarships.

References

[1] D. M. Hamby, "A review of techniques for parameter sensitivity analysis of environmental models," *Environmental Monitoring and Assessment*, vol. 32, no. 2, pp. 135–154, 1994.

[2] T. Homma and A. Saltelli, "Importance measures in global sensitivity analysis of nonlinear models," *Reliability Engineering & System Safety*, vol. 52, no. 1, pp. 1–17, 1996.

[3] A. Saltelli, S. Tarantola, and K. P. S. Chan, "A quantitative model-independent method for global sensitivity analysis of model output," *Technometrics*, vol. 41, no. 1, pp. 39–56, 1999.

[4] R. L. Iman and J. C. Helton, "An investigation of uncertanty and sensitivity analysis techniques for computer-models," *Risk Analysis*, vol. 8, no. 1, pp. 71–90, 1988.

[5] S. Marino, I. B. Hogue, C. J. Ray, and D. E. Kirschner, "A methodology for performing global uncertainty and sensitivity analysis in systems biology," *Journal of Theoretical Biology*, vol. 254, no. 1, pp. 178–196, 2008.

[6] A. Saltelli and P. Annoni, "How to avoid a perfunctory sensitivity analysis," *Environmental Modelling & Software*, vol. 25, no. 12, pp. 1508–1517, 2010.

[7] H. C. Frey and S. R. Patil, "Identification and review of sensitivity analysis methods," *Risk Analysis*, vol. 22, no. 3, pp. 553–578, 2002.

[8] R. L. Iman and W. J. Conover, "Small sample sensitivity analysis techniques for computer-models, with an application to risk assessment," *Communications in Statistics Part A - Theory and Methods*, vol. 9, no. 17, pp. 1749–1842, 1980.

[9] A. Charnes, W. W. Cooper, and B. Mellon, "A model for programming and sensitivity analysis in an integrated oil company," *Econometrica*, vol. 22, no. 2, pp. 193–217, 1954.

[10] D. E. W. Schumann and R. A. Bradley, "The comparison of the sensitivities of similar experiments - model ii of the analysis of variance," *Biometrics*, vol. 15, no. 3, pp. 405–416, 1959.

[11] A. Golub and F. E. Grubbs, "Analysis of sensitivity experiments when the levels of stimulus cannot be controlled," *Journal of the American Statistical Association*, vol. 51, no. 274, pp. 257–265, 1956.

[12] J. C. Helton, J. D. Johnson, C. J. Sallaberry, and C. B. Storlie, "Survey of sampling-based methods for uncertainty and sensitivity analysis," *Reliability Engineering & System Safety*, vol. 91, no. 10-11, pp. 1175–1209, 2006.

[13] J. B. Reitsma, A. S. Glas, A. W. S. Rutjes, R. J. P. M. Scholten, P. M. Bossuyt, and A. H. Zwinderman, "Bivariate analysis of sensitivity and specificity produces informative summary measures in diagnostic reviews," *Journal of Clinical Epidemology*, vol. 58, no. 10, pp. 982–990, 2005.

[14] G. Allaire, F. Jouve, and A. M. Toader, "Structural optimization using sensitivity analysis and a level-set method," *Journal of Computational Physics*, vol. 194, no. 1, pp. 363–393, 2004.

[15] A. van Griensven, T. Meixner, S. Grunwald, T. Bishop, A. Diluzio, and R. Srinivasan, "A global sensitivity analysis tool for the parameters of multi-variable catchment models," *Journal of Hidrology*, vol. 324, no. 1-4, pp. 10–23, 2006.

[16] A. B. Guenther, P. R. Zimmerman, P. C. Harley, R. K. Monson, and R. Fall, "Isoprene and monoterpene emission rate variability: model evaluations and sensitivity analyses," *Journal of Geophysical Research Atmospheres*, vol. 98, no. D7, pp. 12609–12617, 1993.

[17] Y. Zhang, M. A. Dube, D. D. McLean, and M. Kates, "Biodiesel production from waste cooking oil: 2. economic assessment and sensitivity analysis," *Bioresource Technology*, vol. 90, no. 3, pp. 229–240, 2003.

[18] G. Thompson, R. M. Rasmussen, and K. Manning, "Explicit forecasts of winter precipitation using an improved bulk microphysics scheme. part i: Description and sensitivity analysis," *Monthly Weather Review*, vol. 132, no. 2, pp. 519–542, 2004.

[19] D. J. Pannel, "Sensitivity analysis: strategies, methods, concepts, examples," *University of Western Australia, Crawley*, 2009.

[20] D. Bertsimas and M. Sim, "The price of robustness," *Operations Research*, vol. 52, no. 1, pp. 35–53, 2004.

[21] R. Levine and D. Renelt, "A sensitivity analysis of cross-country growth regressions," *American Economic Review*, vol. 82, no. 4, pp. 942–963, 1992.

[22] H. Hegre and N. Sambanis, "Sensitivity analysis of empirical results on civil war onset," *Journal of Conflict Resolution*, vol. 50, no. 4, pp. 508–535, 2006.

[23] T. G. Eschenbach and L. S. McKeague, "Exposition on using graphs for sensitivity analysis," *The Engineering Economist*, vol. 34, no. 4, pp. 313–333, 1989.

[24] J. P. Saraiva, B. S. Lima, V. M. Gomes, P. H. R. Flores, F. A. Gomes, A. O. Assis, M. R. C. Reis, W. R. H. Araujo, C. Abrenhosa, and W. P. Calixto, "Calculation of sensitivity index using one-at-a-time measures based on graphical analysis," *18th International Scientific Conference on Eletric Power Engineering (EPE*, 2017.

[25] A. S. Magalhães, P. H. F. Moraes, P. H. G. Gomes, A. J. Alves, and C. W. P., "Reconditioning in synchronous operation with one parallel induction generator," *Transactions on Environment and Electrical Engineering*, vol. 1, pp. 66–72, 2016.